Solution to 316 Final Examination Su06

1. Using the direct-substitution method and a sampling rate of $f_s = 10$, find the transfer function of a discrete-time filter which approximates a continuous-time system with a transfer function

$$\mathrm{H}(s) = \frac{3s(s+1)}{s^2 + 5s + 6}.$$

The transfer function can be written in the form $\mathrm{H}(z) = A\frac{z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0}$. Find the numerical values of *A*, a_1, a_0, b_1 and b_0.

A = 3 , a_1 = -1.5595 , a_0 = 0.6065 , b_1 = -1.905 , b_0 = 0.905

$$f_s = 10 \Rightarrow T_s = 1/10$$

$$\mathrm{H}(s) = \frac{3s(s+1)}{s^2 + 5s + 6} = 3\frac{(s-0)(s+1)}{(s+3)(s+2)} \Rightarrow \mathrm{H}(z) = 3\frac{(z-1)\left(z - e^{-1/10}\right)}{\left(z - e^{-3/10}\right)\left(z - e^{-1/5}\right)}$$

$$\mathrm{H}(z) = 3\frac{(z-1)(z-0.905)}{(z-0.7408)(z-0.8187)} = 3\frac{z^2 - 1.905z + 0.905}{z^2 - 1.5595z + 0.6065}$$

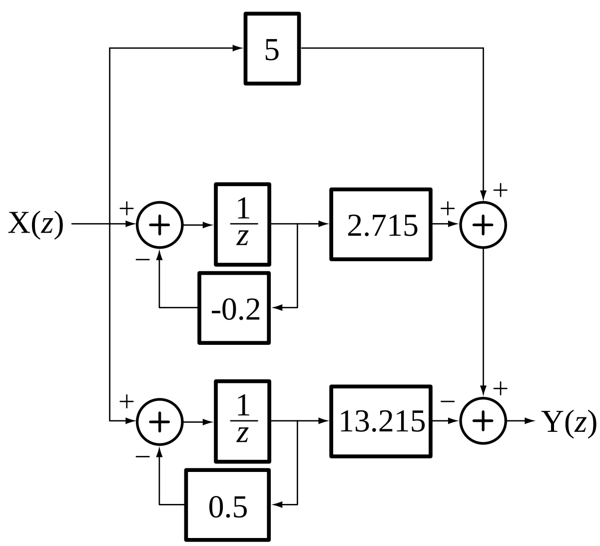
2. Draw a parallel realization of the transfer function

$$H(z) = 5\frac{z^2 - 1.8z + 0.7}{z^2 + 0.3z - 0.1}.$$

$$H(z) = 5\frac{z^2 - 1.8z + 0.7}{z^2 + 0.3z - 0.1} = 5\left(1 - \frac{2.1z - 0.8}{z^2 + 0.3z - 0.1}\right)$$

$$\begin{array}{r|l} & 1 \\ z^2 + 0.3z - 0.1 & z^2 - 1.8z + 0.7 \\ & \underline{z^2 + 0.3z - 0.1} \\ & -2.1z + 0.8 \end{array}$$

$$H(z) = 5\left(1 - \frac{2.1z - 0.8}{(z+0.5)(z-0.2)}\right) = 5\left(1 - \frac{2.643}{z+0.5} + \frac{0.543}{z-0.2}\right) = 5 - \frac{13.215}{z+0.5} + \frac{2.715}{z-0.2}$$



3. Indicate for each of the following transfer functions whether the system is

Stable (S)

Marginally stable (therefore also unstable) (MS)

or

Unstable without being marginally stable (US).

(a) $\mathrm{H}(s) = \dfrac{3s^2 + s + 8}{2s^2 + 7s + 10}$ **[S]** MS US

Poles at $-1.75 \pm j1.3919$. In the open left half-plane. Therefore stable.

(b) $\mathrm{H}(z) = \dfrac{3z^2 + z + 8}{2z^2 + 7z + 10}$ S MS **[US]**

Poles at $-1.75 \pm j1.3919$ or $2.236e^{\pm j2.4697}$. Outside the unit circle. Therefore unstable.

(c) A continuous-time feedback system with forward-path transfer function $\mathrm{H}_1(s) = \dfrac{s^2 + 5s + 10}{s^2 + 2s + 3}$ and feedback-path transfer function $\mathrm{H}_2(s) = -1$.

[S] MS US

$$\mathrm{H}(s) = \frac{\dfrac{s^2 + 5s + 10}{s^2 + 2s + 3}}{1 - \dfrac{s^2 + 5s + 10}{s^2 + 2s + 3}} = \frac{s^2 + 5s + 10}{-3s - 7} = -\frac{s^2 + 5s + 10}{3s + 7} \Rightarrow \text{One pole at } s = -7/3 \Rightarrow \text{Stable}$$

(d) A discrete-time feedback system with forward-path transfer function $\mathrm{H}_1(z) = \dfrac{z^2 + 5z + 10}{z^2 + 2z + 3}$ and feedback-path transfer function $\mathrm{H}_2(z) = -1$.

S MS **[US]**

$$\mathrm{H}(z) = \frac{\dfrac{z^2 + 5z + 10}{z^2 + 2z + 3}}{1 - \dfrac{z^2 + 5z + 10}{z^2 + 2z + 3}} = \frac{z^2 + 5z + 10}{-3z - 7} = -\frac{z^2 + 5z + 10}{3z + 7} \Rightarrow \text{One pole at } z = -7/3 \Rightarrow \text{Unstable}$$

4. An FIR filter is designed with an impulse response $h[n] = 8(u[n] - u[n-3])$.

(a) How many poles and zeros does its transfer function have and where are they located?

$\underline{2}$ Zeros at $\underline{-0.5 \pm j0.866}$
$\underline{2}$ Poles at $\underline{0}$

From the definition of the *z* transform,

$$H(z) = 8\sum_{n=0}^{\infty}(u[n] - u[n-3])z^{-n} = 8(1 + z^{-1} + z^{-2})$$

$$H(z) = 8(1 + 1/z + 1/z^2) = 8\frac{z^2 + z + 1}{z^2}$$

Zeros at $z^2 + z + 1 = 0 \Rightarrow z = -0.5 \pm j0.866$ (which are on the unit circle)

Poles at $z^2 = 0 \Rightarrow$ Double pole at $z = 0$

Alternate Solution:

$$u[n] \xleftrightarrow{z} \frac{z}{z-1} \Rightarrow u[n] - u[n-3] \xleftrightarrow{z} \frac{z}{z-1} - \frac{z^{-2}}{z-1} = z^{-2}\frac{z^3 - 1}{z-1}$$

$$\begin{array}{r|l} & z^2 + z + 1 \\ \hline z-1 & z^3 - 1 \\ & \underline{z^3 - z^2} \\ & z^2 - 1 \\ & \underline{z^2 - z} \\ & z - 1 \\ & \underline{z - 1} \\ & 0 \end{array}$$

$$u[n] \xleftrightarrow{z} \frac{z^2 + z + 1}{z^2},$$

The rest of the solution is as above.

(b) At what values of Ω would the filter response be absolutely zero?

$$e^{j\Omega} = -0.5 \pm j0.866 \Rightarrow \Omega = \pm 2\pi / 3$$