Solution to EECS Final Examination S08 #1

1. In the circuit below let $R_f = 10\text{k}\Omega$ and $C_i = 20\text{nF}$ and let the transfer function be $\mathrm{H}(f) = \dfrac{\mathrm{V}_o(f)}{\mathrm{V}_i(f)}$.

(a) At what numerical cyclic frequency f is the magnitude of $\mathrm{H}(f)$ the smallest?

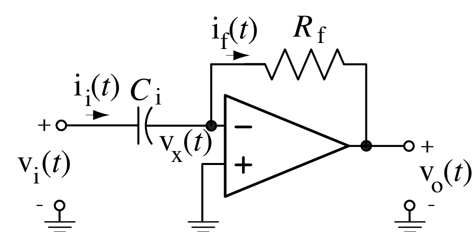
$$\mathrm{H}(f) = -\frac{R_f}{1/j2\pi f C_i} = -j2\pi f R_f C_i = -j2\pi f \times 2 \times 10^{-4}$$

$|\mathrm{H}(f)|$ is a minimum at $f = 0$

(b) Find the numerical value of $|\mathrm{H}(1000)|$.

$$\mathrm{H}(1000) = -j(1000) \times 4\pi \times 10^{-4} = -j0.4\pi = -j1.257$$

$$|\mathrm{H}(1000)| = 1.257$$



2. The discrete Fourier transform, defined by

$$\mathrm{x}[n] = \frac{1}{N_F} \sum_{k=0}^{N_F-1} \mathrm{X}[k] e^{j2\pi kn/N_F} \xleftrightarrow{\mathscr{F}} \mathrm{X}[k] = \sum_{n=0}^{N_F-1} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

takes a set of samples from a time-domain signal $\{\mathrm{x}[0], \mathrm{x}[1], \mathrm{x}[2], \mathrm{x}[3]\}$ and returns the set $\{\mathrm{X}[0], \mathrm{X}[1], \mathrm{X}[2], \mathrm{X}[3]\}$. If $\mathrm{x}[0] = 3$, $\mathrm{x}[1] = -5$, $\mathrm{x}[3] = 9$ and $\mathrm{X}[2] = 6$ find these numerical values. (Be sure to notice whether the x's are lower case or upper case.)

(a) $$\mathrm{X}[2] = 6 = \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n} = 3 + 5 + \mathrm{x}[2] - 9$$

$$\mathrm{x}[2] = -3 - 5 + 9 + 6 = 7$$

(b) $$\mathrm{X}[0] = \sum_{n=0}^{3} \mathrm{x}[n] = 3 - 5 + 7 + 9 = 14$$

(c) $$\mathrm{X}[3] = \sum_{n=0}^{3} \mathrm{x}[n] e^{-j3\pi n/2} = 3 - j5 - 7 - j9 = -4 - j14$$

3. Below is a block diagram of a moving-average filter. Let $N = 4$ and let $x[n] = u[n] - u[n-3]$.

(a) Find the numerical value of $y[5]$.

$$y[n] = (1/4) \sum_{m=n-3}^{n} x[m] \Rightarrow y[5] = (1/4) \sum_{m=2}^{5} \left(u[m] - u[m-3] \right)$$

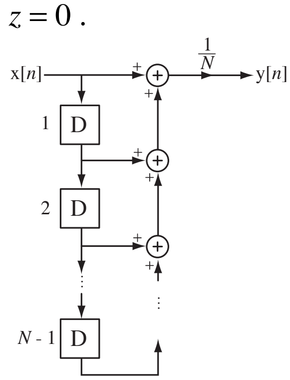
$$y[5] = (1/4) \left(\sum_{m=2}^{5} u[m] - \sum_{m=2}^{5} u[m-3] \right) = (1/4)[4-3] = 1/4$$

(b) How many poles are there in the transfer function $H(z) = \dfrac{Y(z)}{X(z)}$ of this filter and where are they located?

$$h[n] = (1/4)\left(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\right)$$

$$H(z) = (1/4)\left(1 + z^{-1} + z^{-2} + z^{-3}\right) = (1/4)\frac{z^3 + z^2 + z + 1}{z^3}$$

Three poles, all at $z = 0$.



4. For each continuous-time filter, find the numerical cyclic frequency (or frequencies)
f at which its frequency response magnitude is minimum and maximum. (Some answers could be "infinity".)

(a) One finite pole at $s = -3$ and no finite zeros.

$$\mathrm{H}(s) = \frac{K}{s+3} \Rightarrow |\mathrm{H}(j\omega)| = \frac{K}{|j\omega + 3|}$$

Maximum at $f = \omega / 2\pi = 0$, Minimum at $f = \omega / 2\pi \to \infty$

(b) A double finite pole at $s = -10$ and two finite zeros at $s = \pm j4$.

$$\mathrm{H}(s) = \frac{K(s^2+16)}{(s+10)^2} \Rightarrow |\mathrm{H}(j\omega)| = \frac{K(16-\omega^2)}{\left|(j\omega+10)^2\right|}$$

$$|\mathrm{H}(0)| = \frac{16K}{100} \quad , |\mathrm{H}(j\infty)| = K$$

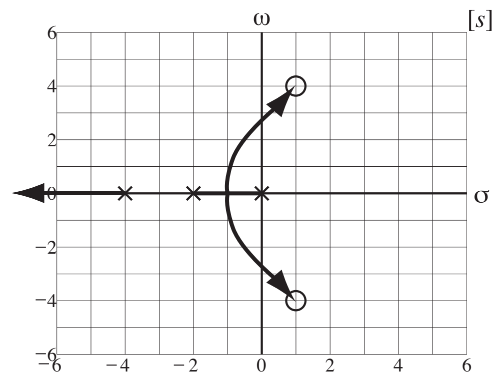
Maximum at $f = \omega / 2\pi \to \infty$, Minimum at $f = \omega / 2\pi = \pm 4 / 2\pi = \pm 2 / \pi = \pm 0.637$

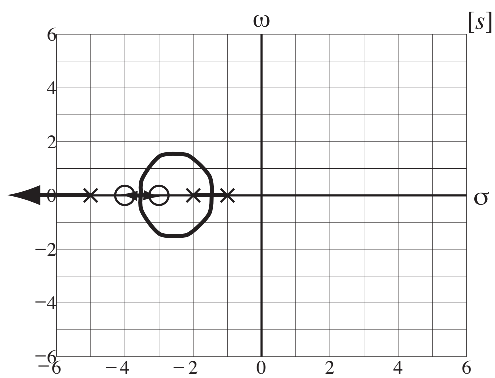
(c) One finite pole at $s = -5$ and one finite zero at $s = +5$.

$$\mathrm{H}(s) = K\frac{s-5}{s+5} \Rightarrow |\mathrm{H}(j\omega)| = K\frac{|j\omega - 5|}{|j\omega + 5|} = K\frac{\sqrt{25+\omega^2}}{\sqrt{25+\omega^2}} = K$$

Maximum is at all frequencies and minimum is also at all frequencies

5. Below are some pole-zero diagrams of the loop transfer function $T(s)$ of two continuous-time feedback systems. Sketch a root locus for each.





6. A digital filter is designed using the impulse invariant method to approximate a continuous-time filter whose transfer function is $\mathrm{H}(s)=\dfrac{1}{s+3}$. The sampling rate is 20 Hz. A unit sequence $\mathrm{u}[n]$ then excites the digital filter. What is the numerical value that the response $\mathrm{y}[n]$ of the digital filter approaches as $n \to \infty$?

$$\mathrm{H}(s)=\frac{1}{s+3} \Rightarrow \mathrm{h}(t)=e^{-3t}\,\mathrm{u}(t)$$

$$\mathrm{h}[n]=e^{-3n/20}\,\mathrm{u}[n]=0.8607^n\,\mathrm{u}[n]$$

The unit sequence response is the accumulation of the unit impulse response. Therefore

$$\mathrm{y}[n]=\sum_{m=-\infty}^{n}0.8607^m\,\mathrm{u}[m]=\sum_{m=0}^{n}0.8607^m=\frac{1-0.8607^{n+1}}{1-0.8607}$$

$$\mathrm{y}[n]_{n\to\infty}=\frac{1}{1-0.8607}=7.179$$

Alternate Solution:

$$\mathrm{H}(z)=\frac{z}{z-0.8607} \Rightarrow \mathrm{H}_{-1}(z)=\frac{z}{z-1}\frac{z}{z-0.8607}=\frac{7.179z}{z-1}-\frac{6.179z}{z-0.8607}$$

$$\mathrm{h}_{-1}[n]=\mathrm{y}[n]=\left[7.179-6.179(0.8607)^n\right]\mathrm{u}[n]$$

$$\mathrm{y}[n]_{n\to\infty}=7.179 \qquad \text{Check.}$$

Solution to EECS Final Examination S08 #2

1. In the circuit below let $R_f = 5\text{k}\Omega$ and $C_i = 20\text{nF}$ and let the transfer function be $\text{H}(f) = \dfrac{\text{V}_o(f)}{\text{V}_i(f)}$.

   (a) At what numerical cyclic frequency f is the magnitude of $\text{H}(f)$ the smallest?

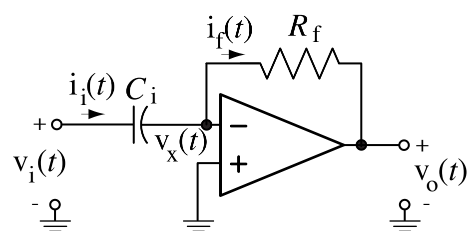
$$\text{H}(f) = -\frac{R_f}{1/j2\pi fC_i} = -j2\pi fR_fC_i = -j2\pi f \times 10^{-4}$$

$$\left|\text{H}(f)\right| \text{ is a minimum at } f = 0$$

   (b) Find the numerical value of $\left|\text{H}(1000)\right|$.

$$\text{H}(1000) = -j(1000) \times 2\pi \times 10^{-4} = -j0.2\pi = -j0.629$$

$$\left|\text{H}(1000)\right| = 0.629$$



2. The discrete Fourier transform, defined by

$$x[n] = \frac{1}{N_F}\sum_{k=0}^{N_F-1} X[k]e^{j2\pi kn/N_F} \xleftrightarrow{\mathcal{F}} X[k] = \sum_{n=0}^{N_F-1} x[n]e^{-j2\pi kn/N_F}$$

takes a set of samples from a time-domain signal $\{x[0],x[1],x[2],x[3]\}$ and returns the set $\{X[0],X[1],X[2],X[3]\}$. If $x[0]=3$, $x[1]=-5$, $x[3]=4$ and $X[2]=3$ find these numerical values. (Be sure to notice whether the x's are lower case or upper case.)

(a) $X[2]=3=\sum_{n=0}^{3} x[n]e^{-j\pi n} = 3+5+x[2]-4$

$$x[2] = -3-5+4+3 = -1$$

(b) $X[0]=\sum_{n=0}^{3} x[n] = 3-5-1+4 = 1$

(c) $X[3]=\sum_{n=0}^{3} x[n]e^{-j3\pi n/2} = 3-j5+1-j4 = 4-j9$

3. Below is a block diagram of a moving-average filter. Let $N = 4$ and let $x[n] = u[n] - u[n-2]$.

(a) Find the numerical value of $y[5]$.

$$y[n] = (1/4) \sum_{m=n-3}^{n} x[m] \Rightarrow y[5] = (1/4) \sum_{m=2}^{5} \left(u[m] - u[m-2]\right)$$

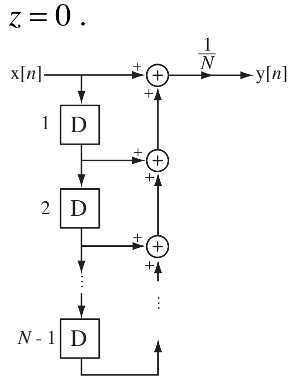
$$y[5] = (1/4)\left(\sum_{m=2}^{5} u[m] - \sum_{m=2}^{5} u[m-2]\right) = (1/4)[4-4] = 0$$

(b) How many poles are there in the transfer function $H(z) = \dfrac{Y(z)}{X(z)}$ of this filter and where are they located?

$$h[n] = (1/4)\left(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\right)$$

$$H(z) = (1/4)\left(1 + z^{-1} + z^{-2} + z^{-3}\right) = (1/4)\frac{z^3 + z^2 + z + 1}{z^3}$$

Three poles, all at $z = 0$.



4. For each continuous-time filter, find the numerical cyclic frequency (or frequencies)
f at which its frequency response magnitude is minimum and maximum. (Some answers could be "infinity".)

(a) One finite pole at $s = -3$ and no finite zeros.

$$\mathrm{H}(s) = \frac{K}{s+3} \Rightarrow |\mathrm{H}(j\omega)| = \frac{K}{|j\omega + 3|}$$

Maximum at $f = \omega / 2\pi = 0$, Minimum at $f = \omega / 2\pi \to \infty$

(b) A double finite pole at $s = -6$ and two finite zeros at $s = \pm j3$.

$$\mathrm{H}(s) = \frac{K(s^2+9)}{(s+6)^2} \Rightarrow |\mathrm{H}(j\omega)| = \frac{K(9-\omega^2)}{|(j\omega+6)^2|}$$

$$|\mathrm{H}(0)| = \frac{9K}{36} \quad , |\mathrm{H}(j\infty)| = K$$

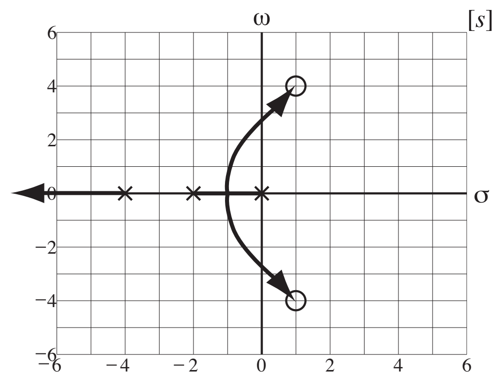
Maximum at $f = \omega / 2\pi \to \infty$, Minimum at $f = \omega / 2\pi = \pm 3 / 2\pi = \pm 0.4775$

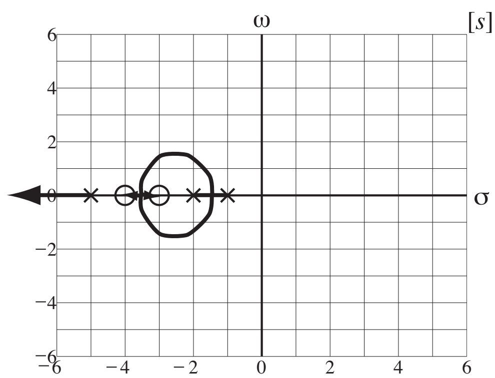
(c) One finite pole at $s = -7$ and one finite zero at $s = +7$.

$$\mathrm{H}(s) = K\frac{s-7}{s+7} \Rightarrow |\mathrm{H}(j\omega)| = K\frac{|j\omega - 7|}{|j\omega + 7|} = K\frac{\sqrt{49+\omega^2}}{\sqrt{49+\omega^2}} = K$$

Maximum is at all frequencies and minimum is also at all frequencies

5. Below are some pole-zero diagrams of the loop transfer function $T(s)$ of two continuous-time feedback systems. Sketch a root locus for each.





6. A digital filter is designed using the impulse invariant method to approximate a continuous-time filter whose transfer function is $\mathrm{H}(s) = \frac{1}{s+3}$. The sampling rate is 10 Hz. A unit sequence $\mathrm{u}[n]$ then excites the digital filter. What is the numerical value that the response $\mathrm{y}[n]$ of the digital filter approaches as $n \to \infty$?

$$\mathrm{H}(s) = \frac{1}{s+3} \Rightarrow \mathrm{h}(t) = e^{-3t}\,\mathrm{u}(t)$$

$$\mathrm{h}[n] = e^{-3n/10}\,\mathrm{u}[n] = 0.7408^n\,\mathrm{u}[n]$$

The unit sequence response is the accumulation of the unit impulse response. Therefore

$$\mathrm{y}[n] = \sum_{m=-\infty}^{n} 0.7408^m\,\mathrm{u}[m] = \sum_{m=0}^{n} 0.7408^m = \frac{1-0.7408^{n+1}}{1-0.7408}$$

$$\mathrm{y}[n]_{n\to\infty} = \frac{1}{1-0.7408} = 3.858$$

Alternate Solution:

$$\mathrm{H}(z) = \frac{z}{z-0.8607} \Rightarrow \mathrm{H}_{-1}(z) = \frac{z}{z-1}\frac{z}{z-0.7408} = \frac{3.858z}{z-1} - \frac{2.858z}{z-0.8607}$$

$$\mathrm{h}_{-1}[n] = \mathrm{y}[n] = \left[3.858 - 2.858(0.7408)^n\right]\mathrm{u}[n]$$

$$\mathrm{y}[n]_{n\to\infty} = 3.858 \qquad \text{Check.}$$

Solution to EECS Final Examination S08 #3

1. In the circuit below let $R_f = 2\text{k}\Omega$ and $C_i = 20\text{nF}$ and let the transfer function be $\text{H}(f) = \dfrac{\text{V}_o(f)}{\text{V}_i(f)}$.

   (a) At what numerical cyclic frequency f is the magnitude of $\text{H}(f)$ the smallest?

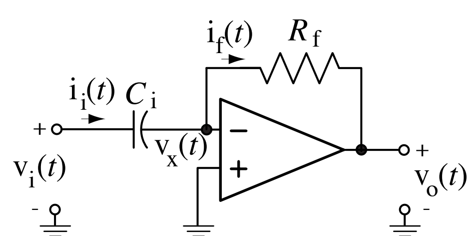
$$\text{H}(f) = -\frac{R_f}{1/j2\pi f C_i} = -j2\pi f R_f C_i = -j2\pi f \times 4 \times 10^{-5}$$

$$\left|\text{H}(f)\right| \text{ is a minimum at } f = 0$$

   (b) Find the numerical value of $\left|\text{H}(1000)\right|$.

$$\text{H}(1000) = -j(1000) \times 8\pi \times 10^{-5} = -j0.08\pi = -j0.2513$$

$$\left|\text{H}(1000)\right| = 0.2513$$



2. The discrete Fourier transform, defined by

$$\mathrm{x}[n] = \frac{1}{N_F}\sum_{k=0}^{N_F-1}\mathrm{X}[k]e^{j2\pi kn/N_F} \xleftrightarrow{\mathcal{F}} \mathrm{X}[k] = \sum_{n=0}^{N_F-1}\mathrm{x}[n]e^{-j2\pi kn/N_F}$$

takes a set of samples from a time-domain signal $\{\mathrm{x}[0],\mathrm{x}[1],\mathrm{x}[2],\mathrm{x}[3]\}$ and returns the set $\{\mathrm{X}[0],\mathrm{X}[1],\mathrm{X}[2],\mathrm{X}[3]\}$. If $\mathrm{x}[0]=3$, $\mathrm{x}[1]=5$, $\mathrm{x}[3]=9$ and $\mathrm{X}[2]=-13$ find these numerical values. (Be sure to notice whether the x's are lower case or upper case.)

(a) $$\mathrm{X}[2] = -13 = \sum_{n=0}^{3}\mathrm{x}[n]e^{-j\pi n} = 3-5+\mathrm{x}[2]-9$$

$$\mathrm{x}[2] = -3+5+9-13 = -2$$

(b) $$\mathrm{X}[0] = \sum_{n=0}^{3}\mathrm{x}[n] = 3+5-2+9 = 15$$

(c) $$\mathrm{X}[3] = \sum_{n=0}^{3}\mathrm{x}[n]e^{-j3\pi n/2} = 3+j5+2-j9 = 5-j4$$

3. Below is a block diagram of a moving-average filter. Let $N = 4$ and let $\mathrm{x}[n] = \mathrm{u}[n] - \mathrm{u}[n-4]$.

(a) Find the numerical value of $\mathrm{y}[5]$.

$$\mathrm{y}[n] = (1/4)\sum_{m=n-3}^{n} \mathrm{x}[m] \Rightarrow \mathrm{y}[5] = (1/4)\sum_{m=2}^{5}\left(\mathrm{u}[m] - \mathrm{u}[m-4]\right)$$

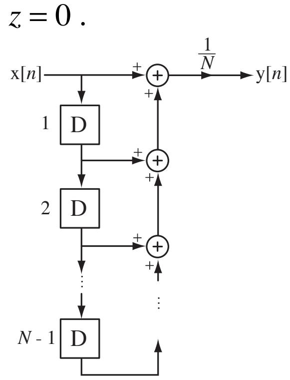
$$\mathrm{y}[5] = (1/4)\left(\sum_{m=2}^{5}\mathrm{u}[m] - \sum_{m=2}^{5}\mathrm{u}[m-4]\right) = (1/4)[4-2] = 1/2$$

(b) How many poles are there in the transfer function $\mathrm{H}(z) = \dfrac{\mathrm{Y}(z)}{\mathrm{X}(z)}$ of this filter and where are they located?

$$\mathrm{h}[n] = (1/4)\left(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\right)$$

$$\mathrm{H}(z) = (1/4)\left(1 + z^{-1} + z^{-2} + z^{-3}\right) = (1/4)\frac{z^3 + z^2 + z + 1}{z^3}$$

Three poles, all at $z = 0$.



4. For each continuous-time filter, find the numerical cyclic frequency (or frequencies)

f at which its frequency response magnitude is minimum and maximum. (Some answers could be "infinity".)

(a) One finite pole at $s=-3$ and no finite zeros.

$$\mathrm{H}(s)=\frac{K}{s+3}\Rightarrow\left|\mathrm{H}(j\omega)\right|=\frac{K}{|j\omega+3|}$$

Maximum at $f=\omega/2\pi=0$, Minimum at $f=\omega/2\pi\rightarrow\infty$

(b) A double finite pole at $s=-8$ and two finite zeros at $s=\pm j2$.

$$\mathrm{H}(s)=\frac{K\left(s^2+4\right)}{(s+8)^2}\Rightarrow\left|\mathrm{H}(j\omega)\right|=\frac{K\left(4-\omega^2\right)}{\left|(j\omega+8)^2\right|}$$

$$\left|\mathrm{H}(0)\right|=\frac{4K}{64}\quad,\left|\mathrm{H}(j\infty)\right|=K$$

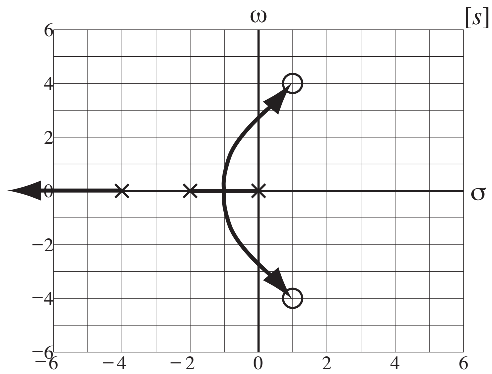
Maximum at $f=\omega/2\pi\rightarrow\infty$, Minimum at $f=\omega/2\pi=\pm2/2\pi=\pm1/\pi=\pm0.3183$

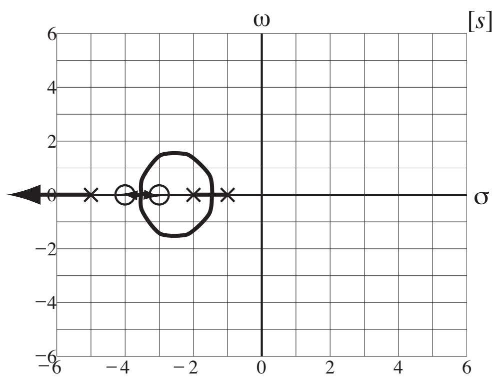
(c) One finite pole at $s=-2$ and one finite zero at $s=+2$.

$$\mathrm{H}(s)=K\frac{s-2}{s+2}\Rightarrow\left|\mathrm{H}(j\omega)\right|=K\frac{|j\omega-2|}{|j\omega+2|}=K\frac{\sqrt{4+\omega^2}}{\sqrt{4+\omega^2}}=K$$

Maximum is at all frequencies and minimum is also at all frequencies

5. Below are some pole-zero diagrams of the loop transfer function $T(s)$ of two continuous-time feedback systems. Sketch a root locus for each.





6. A digital filter is designed using the impulse invariant method to approximate a continuous-time filter whose transfer function is $\mathrm{H}(s)=\frac{1}{s+3}$. The sampling rate is 30 Hz. A unit sequence $\mathrm{u}[n]$ then excites the digital filter. What is the numerical value that the response $\mathrm{y}[n]$ of the digital filter approaches as $n \to \infty$?

$$\mathrm{H}(s)=\frac{1}{s+3} \Rightarrow \mathrm{h}(t)=e^{-3t}\,\mathrm{u}(t)$$

$$\mathrm{h}[n]=e^{-3n/30}\,\mathrm{u}[n]=0.9048^n\,\mathrm{u}[n]$$

The unit sequence response is the accumulation of the unit impulse response. Therefore

$$\mathrm{y}[n]=\sum_{m=-\infty}^{n}0.9048^m\,\mathrm{u}[m]=\sum_{m=0}^{n}0.9048^m=\frac{1-0.9048^{n+1}}{1-0.9048}$$

$$\mathrm{y}[n]_{n\to\infty}=\frac{1}{1-0.9048}=10.504$$

Alternate Solution:

$$\mathrm{H}(z)=\frac{z}{z-0.9048} \Rightarrow \mathrm{H}_{-1}(z)=\frac{z}{z-1}\frac{z}{z-0.9048}=\frac{10.504z}{z-1}-\frac{9.504z}{z-0.9408}$$

$$\mathrm{h}_{-1}[n]=\mathrm{y}[n]=\left[10.504-9.504(0.9408)^n\right]\mathrm{u}[n]$$

$$\mathrm{y}[n]_{n\to\infty}=10.504 \qquad \text{Check.}$$