Solution of ECE 316 Final Examination Su08

1. A continuous-time filter with transfer function

$$H_s(s) = \frac{s+4}{s^2+8s+12}$$

is approximated by a digital filter designed by the bilinear method with a sampling rate of 5 samples/second. Its transfer function is of the form

$$H_z(z) = \frac{b_2 z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0}.$$

Find the numerical values of the constants

$$H_z(z) = \left[\frac{s+4}{s^2+8s+12}\right]_{s \to \frac{2}{T_s}\frac{z-1}{z+1}} = \frac{\frac{2}{T_s}\frac{z-1}{z+1}+4}{\left(\frac{2}{T_s}\frac{z-1}{z+1}\right)^2 + 8 \times \frac{2}{T_s}\frac{z-1}{z+1} + 12}$$

$$H_z(z) = \frac{2T_s(z-1)(z+1) + 4T_s^2(z+1)^2}{4(z-1)^2 + 16T_s(z-1)(z+1) + 12T_s^2(z+1)^2}$$

$$H_z(z) = \frac{2T_s z^2 - 2T_s + 4T_s^2 z^2 + 8T_s^2 z + 4T_s^2}{4z^2 - 8z + 4 + 16T_s z^2 - 16T_s + 12T_s^2 z^2 + 24T_s^2 z + 12T_s^2}$$

$$H_z(z) = \frac{\left(2T_s + 4T_s^2\right)z^2 + 8T_s^2 z + 2T_s\left(2T_s - 1\right)}{\left(4 + 16T_s + 12T_s^2\right)z^2 - \left(8 - 24T_s^2\right)z + 4\left(1 - 4T_s + 3T_s^2\right)}$$

$$f_s = 5 \Rightarrow T_s = 1/5$$

$$H_z(z) = \frac{(14/25)z^2 + (8/25)z - 6/25}{(192/25)z^2 - (176/25)z + 32/25} = \frac{(7/96)z^2 + (1/24)z - 1/32}{z^2 - (11/12)z + 1/6}$$

b_2 = 7/96 or 0.0729 , b_1 = 1/24 or 0.0417 , b_0 = -1/32 or -0.0312

a_1 = -11/12 or –0.9167 , a_0 = 1/6 or 0.1667

2. A continuous-time filter with transfer function

$$\mathrm{H}_s(s) = \frac{5}{s(s+8)}$$

is approximated by a digital filter designed by the impulse invariant method with a sampling rate of 20 samples/second. Its transfer function is of the form

$$\mathrm{H}_z(z) = \frac{b_2 z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0} .$$

Find the numerical values of the constants.

$$\mathrm{H}_s(s) = \frac{5}{s(s+8)} = (5/8)\left(\frac{1}{s} - \frac{1}{s+8}\right) \Rightarrow \mathrm{h}(t) = (5/8)\left(1 - e^{-8t}\right)\mathrm{u}(t)$$

$$\mathrm{h}[n] = (5/8)\left(1 - e^{-8n/20}\right)\mathrm{u}[n] = (5/8)\left(1 - 0.6703^n\right)\mathrm{u}[n]$$

$$\mathrm{H}_z(z) = (5/8)\left(\frac{z}{z-1} - \frac{z}{z-0.6703}\right) = \frac{5}{8} \times \frac{z^2 - 0.6703z - z^2 + z}{z^2 - 1.6703z + 0.6703} = \frac{0.2060z}{z^2 - 1.6703z + 0.6703}$$

b_2 = $\underline{0}$, b_1 = $\underline{0.2060}$, b_0 = $\underline{0}$

a_1 = $\underline{-1.6703}$, a_0 = $\underline{0.6703}$

3. Each pole-zero diagram below represents the poles and zeros of the loop transfer function ($T(s)$) of a feedback system. Sketch a root locus on each diagram.

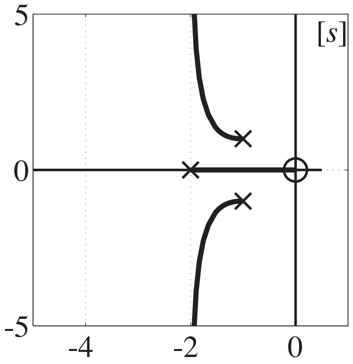
Two of these systems will become unstable at a finite, non-zero value of the gain factor "*K*" in the loop transfer function. Which ones?

Systems that will go unstable for a finite, non-zero *K* B I

Three of these systems are marginally stable for an infinite *K*. Which ones?

Marginally stable systems for an infinite *K* A C G

A



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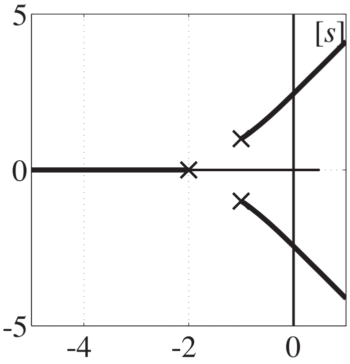

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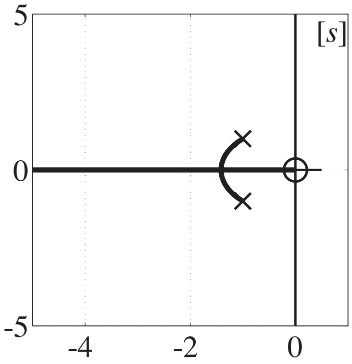

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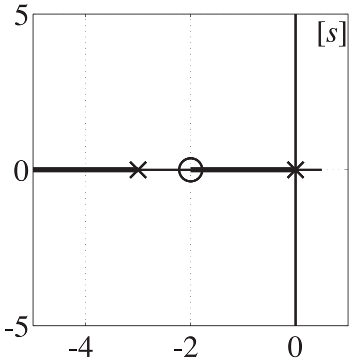

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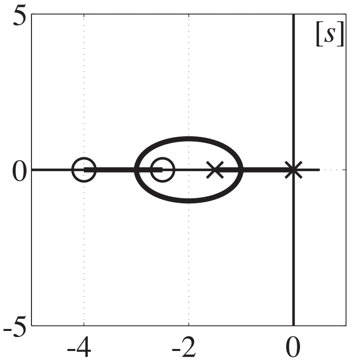

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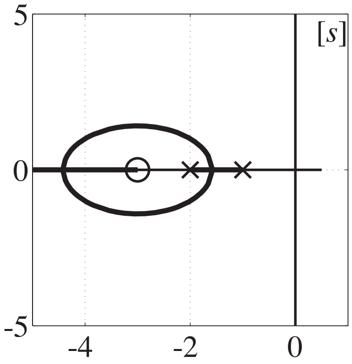

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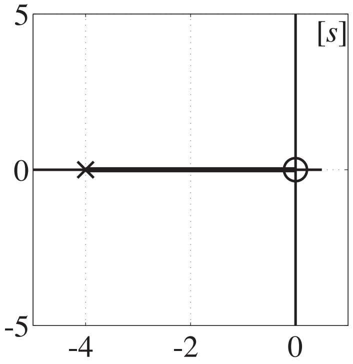

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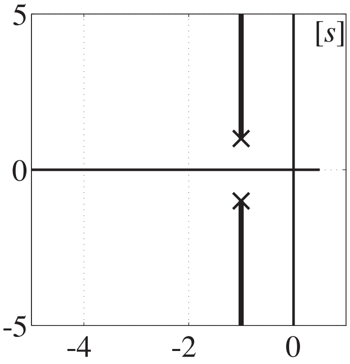

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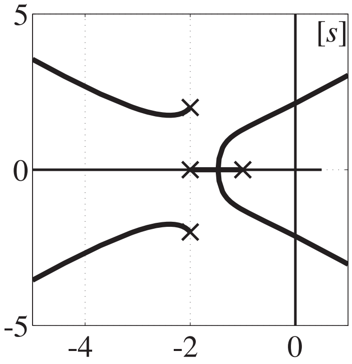
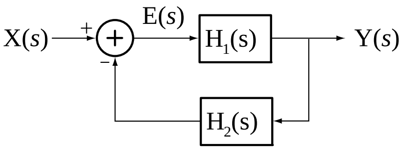


4. In the general feedback system below let $\mathrm{H}_2(s) = 1$. Then choose the steady-state error between the excitation and the response as either zero, finite or infinite (by circling it) for each case of $\mathrm{H}_1(s)$ and $\mathrm{x}(t)$.



			Steady-state Error
(a)	$\mathrm{H}_1(s) = 10/s$	$\mathrm{x}(t) = \mathrm{u}(t)$	[✓ Zero] Finite Infinite
(b)	$\mathrm{H}_1(s) = \frac{10s}{s+3}$	$\mathrm{x}(t) = \mathrm{u}(t)$	Zero [✓ Finite] Infinite
(c)	$\mathrm{H}_1(s) = \frac{10}{s^2 + 3s}$	$\mathrm{x}(t) = \mathrm{u}(t)$	[✓ Zero] Finite Infinite
(d)	$\mathrm{H}_1(s) = \frac{10}{s^2 + 3s}$	$\mathrm{x}(t) = \mathrm{ramp}(t)$	Zero [✓ Finite] Infinite
(e)	$\mathrm{H}_1(s) = \frac{10}{s^2 + 3s + 5}$	$\mathrm{x}(t) = \mathrm{ramp}(t)$	Zero Finite [✓ Infinite]
(f)	$\mathrm{H}_1(s) = 10/s^2$	$\mathrm{x}(t) = \mathrm{ramp}(t)$	[✓ Zero] Finite Infinite

5. The loop transfer function of a discrete-time feedback system has more finite poles than finite zeros. Will it go unstable at a finite value of the gain factor "*K*"? Explain your answer.

[✓ Will go unstable]   Will not go unstable

Explanation

Will go unstable because at least one branch of the root locus must terminate on a zero at infinity and therefore go outside the unit circle.

6. A sinusoidal signal of frequency f_0 is impulse sampled at a rate of f_s. The impulse-sampled signal is the excitation of an ideal lowpass filter with corner (cutoff) frequency of f_c. For each set of parameters below what frequencies are present in the response of the filter? (List only the non-negative frequencies, including zero if present.)

In each case the frequencies present in the filter excitation are $kf_s \pm f_0$ where *k* is any integer. Only those whose absolute values are below the lowpass filter's cutoff frequency appear in the response.

(a) $f_0 = 20$, $f_s = 50$, $f_c = 210$

Frequencies are 20, 30, 70, 80, 120, 130, 170, 180

(b) $f_0 = 20$, $f_s = 50$, $f_c = 40$

Frequencies are 20, 30

(c) $f_0 = 20$, $f_s = 15$, $f_c = 35$

Frequencies are 5, 10, 20, 25, 35

(d) $f_0 = 20$, $f_s = 5$, $f_c = 22$

Frequencies are 0, 5, 10, 15, 20

(e) $f_0 = 20$, $f_s = 20$, $f_c = 50$

Frequencies are 0, 20, 40

Solution of ECE 316 Final Examination Su08

1. A continuous-time filter with transfer function

$$\mathrm{H}_s\left(s\right)=\frac{s+4}{s^2+8s+12}$$

is approximated by a digital filter designed by the bilinear method with a sampling rate of 8 samples/second. Its transfer function is of the form

$$\mathrm{H}_z\left(z\right)=\frac{b_2z^2+b_1z+b_0}{z^2+a_1z+a_0}.$$

Find the numerical values of the constants

$$\mathrm{H}_z\left(z\right)=\left[\frac{s+4}{s^2+8s+12}\right]_{s\to\frac{2}{T_s}\frac{z-1}{z+1}}=\frac{\frac{2}{T_s}\frac{z-1}{z+1}+4}{\left(\frac{2}{T_s}\frac{z-1}{z+1}\right)^2+8\times\frac{2}{T_s}\frac{z-1}{z+1}+12}$$

$$\mathrm{H}_z\left(z\right)=\frac{2T_s\left(z-1\right)\left(z+1\right)+4T_s^2\left(z+1\right)^2}{4\left(z-1\right)^2+16T_s\left(z-1\right)\left(z+1\right)+12T_s^2\left(z+1\right)^2}$$

$$\mathrm{H}_z\left(z\right)=\frac{2T_sz^2-2T_s+4T_s^2z^2+8T_s^2z+4T_s^2}{4z^2-8z+4+16T_sz^2-16T_s+12T_s^2z^2+24T_s^2z+12T_s^2}$$

$$\mathrm{H}_z\left(z\right)=\frac{\left(2T_s+4T_s^2\right)z^2+8T_s^2z+2T_s\left(2T_s-1\right)}{\left(4+16T_s+12T_s^2\right)z^2-\left(8-24T_s^2\right)z+4\left(1-4T_s+3T_s^2\right)}$$

$$f_s=8\Rightarrow T_s=1/8$$

$$\mathrm{H}_z\left(z\right)=\frac{\left(5/16\right)z^2+\left(1/8\right)z-3/16}{\left(99/16\right)z^2-\left(61/8\right)z+140/64}=\frac{\left(5/99\right)z^2+\left(2/99\right)z-3/99}{z^2-\left(122/99\right)z+140/396}$$

b_2 = 5/99 or 0.0505 , b_1 = 2/99 or 0.0202 , b_0 = -3/99 or -0.0303
a_1 = -122/99 or –1.2323 , a_0 = 140/396 or 0.3535

2. A continuous-time filter with transfer function

$$\mathrm{H}_s(s) = \frac{5}{s(s+8)}$$

is approximated by a digital filter designed by the impulse invariant method with a sampling rate of 12 samples/second. Its transfer function is of the form

$$\mathrm{H}_z(z) = \frac{b_2 z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0}.$$

Find the numerical values of the constants.

$$\mathrm{H}_s(s) = \frac{5}{s(s+8)} = (5/8)\left(\frac{1}{s} - \frac{1}{s+8}\right) \Rightarrow \mathrm{h}(t) = (5/8)\left(1 - e^{-8t}\right)\mathrm{u}(t)$$

$$\mathrm{h}[n] = (5/8)\left(1 - e^{-8n/12}\right)\mathrm{u}[n] = (5/8)\left(1 - 0.5134^n\right)\mathrm{u}[n]$$

$$\mathrm{H}_z(z) = (5/8)\left(\frac{z}{z-1} - \frac{z}{z-0.5134}\right) = \frac{5}{8} \times \frac{z^2 - 0.5134z - z^2 + z}{z^2 - 1.5134z + 0.5134} = \frac{0.304z}{z^2 - 1.5134z + 0.5134}$$

$b_2 = \underline{0}$, $b_1 = \underline{0.304}$, $b_0 = \underline{0}$

$a_1 = \underline{-1.5134}$, $a_0 = \underline{0.5134}$

3. Each pole-zero diagram below represents the poles and zeros of the loop transfer function ($T(s)$) of a feedback system. Sketch a root locus on each diagram.

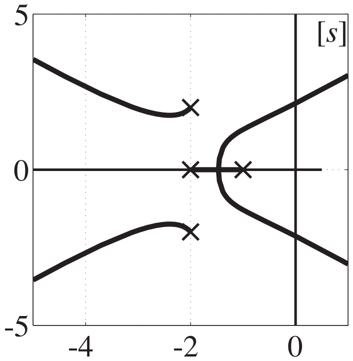
Two of these systems will become unstable at a finite, non-zero value of the gain factor "*K*" in the loop transfer function. Which ones?

Systems that will go unstable for a finite, non-zero *K* A D

Three of these systems are marginally stable for an infinite *K*. Which ones?

Marginally stable systems for an infinite *K* B C G

A



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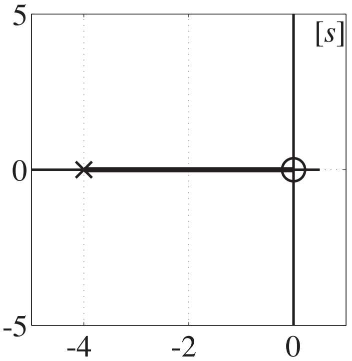

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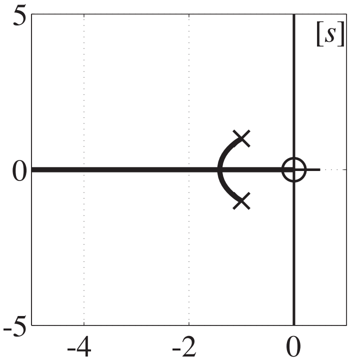

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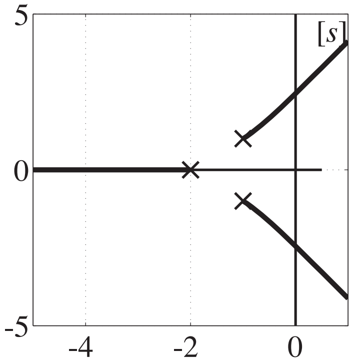

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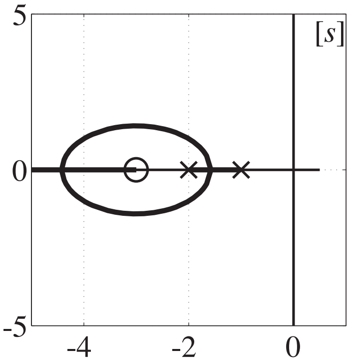

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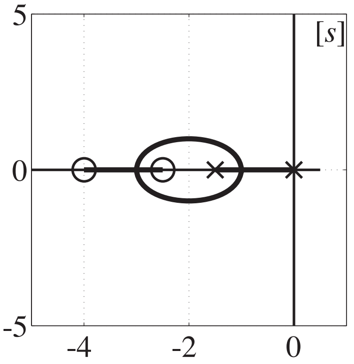

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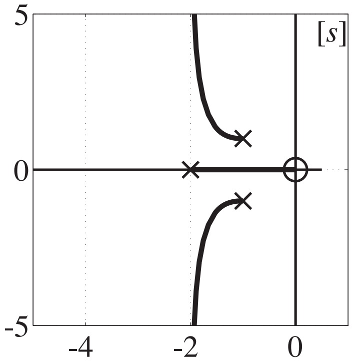

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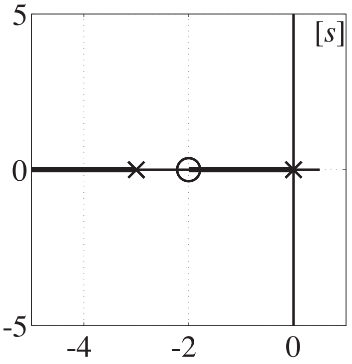

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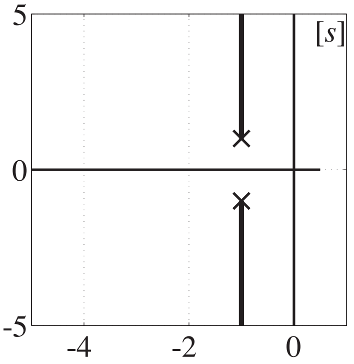
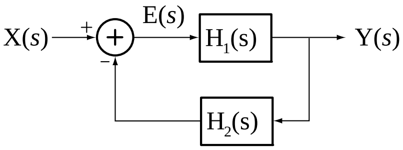


4. In the general feedback system below let $\mathrm{H}_2(s) = 1$. Then choose the steady-state error between the excitation and the response as either zero, finite or infinite (by circling it) for each case of $\mathrm{H}_1(s)$ and $\mathrm{x}(t)$.



			Steady-state Error
(a)	$\mathrm{H}_1(s) = \frac{10}{s^2 + 3s}$	$\mathrm{x}(t) = \mathrm{ramp}(t)$	Zero [✓ Finite] Infinite
(b)	$\mathrm{H}_1(s) = \frac{10}{s^2 + 3s + 5}$	$\mathrm{x}(t) = \mathrm{ramp}(t)$	Zero Finite [✓ Infinite]
(c)	$\mathrm{H}_1(s) = 10 / s^2$	$\mathrm{x}(t) = \mathrm{ramp}(t)$	[✓ Zero] Finite Infinite
(d)	$\mathrm{H}_1(s) = 10 / s$	$\mathrm{x}(t) = \mathrm{u}(t)$	[✓ Zero] Finite Infinite
(e)	$\mathrm{H}_1(s) = \frac{10s}{s + 3}$	$\mathrm{x}(t) = \mathrm{u}(t)$	Zero [✓ Finite] Infinite
(f)	$\mathrm{H}_1(s) = \frac{10}{s^2 + 3s}$	$\mathrm{x}(t) = \mathrm{u}(t)$	[✓ Zero] Finite Infinite

5. The loop transfer function of a discrete-time feedback system has more finite poles than finite zeros. Will it go unstable at a finite value of the gain factor "K"? Explain your answer.

[✓ Will go unstable]   Will not go unstable (1 pt)

Explanation

Will go unstable because at least one branch of the root locus must terminate on a zero at infinity and therefore go outside the unit circle.

6. A sinusoidal signal of frequency f_0 is impulse sampled at a rate of f_s. The impulse-sampled signal is the excitation of an ideal lowpass filter with corner (cutoff) frequency of f_c. For each set of parameters below what frequencies are present in the response of the filter? (List only the non-negative frequencies, including zero if present.) (3 pts each)

In each case the frequencies present in the filter excitation are $kf_s \pm f_0$ where k is any integer. Only those whose absolute values are below the lowpass filter's cutoff frequency appear in the response.

(a) $f_0 = 20$, $f_s = 45$, $f_c = 210$

Frequencies are 20, 25, 65, 70, 110, 115, 155, 160, 200, 195, 205

(b) $f_0 = 20$, $f_s = 45$, $f_c = 40$

Frequencies are 20, 25

(c) $f_0 = 20$, $f_s = 12$, $f_c = 35$

Frequencies are 4,8,16, 20, 28, 32

(d) $f_0 = 20$, $f_s = 3$, $f_c = 9$

Frequencies are 1,2,4,5,7,8

(e) $f_0 = 30$, $f_s = 30$, $f_c = 70$

Frequencies are 0, 30, 60