Solution of ECE 316 Final Examination S09

1. A second-order analog bandpass filter with a transfer function of the general form

$$\mathrm{H}(s) = \frac{Ks}{s^2 + as + b}$$

is approximated by a digital filter designed using the bilinear method.

(a) How many zeros will the digital filter have?

(b) Knowing only what this problem states, is it possible to locate the zeros numerically?

(c) If it is possible to locate the zeros numerically, where are they?

(d) How many poles will the digital filter have?

(b) Knowing only what this problem states, is it possible to locate the poles numerically?

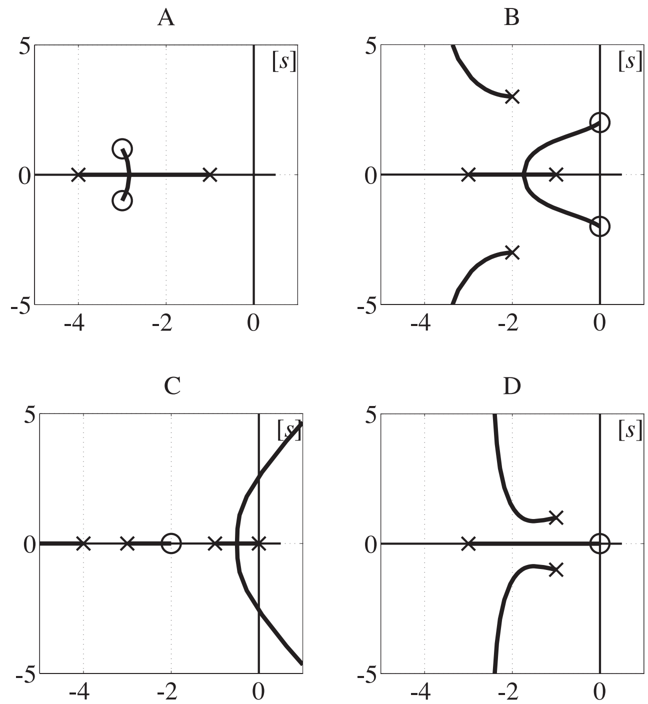
(c) If it is possible to locate the poles numerically, where are they?

$$\mathrm{H}_a(s) = \frac{Ks}{s^2 + bs + c} \Rightarrow \mathrm{H}_d(z) = \frac{K\frac{2}{T_s}\frac{z-1}{z+1}}{\left(\frac{2}{T_s}\frac{z-1}{z+1}\right)^2 + b\frac{2}{T_s}\frac{z-1}{z+1} + c}$$

$$\mathrm{H}_d(z) = \frac{2KT_s(z+1)(z-1)}{4(z-1)^2 + 2bT_s(z+1)(z-1) + cT_s^2(z+1)^2}$$

Zeros at $z = \pm 1$. Two poles. Cannot locate the poles numerically with this information.

2. Below are pole/zero diagrams for the loop transfer functions of some feedback systems. Draw a root locus for each one.



3. A discrete time feedback system has a forward path transfer function $\mathrm{H}_1(z) = \frac{Kz}{z+0.4}$ and a feedback path transfer function $\mathrm{H}_2(z) = 1$. For what numerical range of K's is this feedback system stable?

$$\mathrm{H}(z) = \frac{\frac{Kz}{z+0.4}}{1+\frac{Kz}{z+0.4}} = \frac{Kz}{z(1+K)+0.4}$$

$$\text{Poles at } z(1+K)+0.4 = 0 \Rightarrow z = -\frac{0.4}{1+K}$$

$$\text{For stability, } \left|-\frac{0.4}{1+K}\right| < 1 \Rightarrow K > -0.6 \text{ or } K < -1.4$$

4. What are the numerical locations of the poles and zeros of a 2nd order butterworth highpass filter with a corner radian frequency of $\omega = 10$? (Specify the pole and zero locations in the form $A\angle B$ where A is the magnitude and B is the angle in radians. For example, $6\angle 1.4$.)

$$n = 2 \Rightarrow \text{Normalized lowpass poles at } e^{j3\pi/4}\ ,\ e^{-j3\pi/4} \Rightarrow \mathrm{H}_{LP}(s) = \frac{1}{\left(s - \frac{-1+j}{\sqrt{2}}\right)\left(s - \frac{-1-j}{\sqrt{2}}\right)}$$

$$\mathrm{H}_{LP}(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \Rightarrow \mathrm{H}_{HP}(s) = \left[\frac{1}{s^2 + \sqrt{2}s + 1}\right]_{s \to 10/s} = \frac{1}{(10/s)^2 + 10\sqrt{2}/s + 1} = \frac{s^2}{s^2 + 10\sqrt{2}s + 100}$$

Two zeros at $s = 0$ and Poles at $s = \dfrac{-10\sqrt{2} \pm \sqrt{200 - 400}}{2} = -5\sqrt{2} \pm j\sqrt{50} = 10\angle 2.3562$ and $10\angle -2.3562$

5. Find the inverse Laplace transforms of these signals.

(a) $\mathrm{H}(s) = \dfrac{3s^2}{s^2 + 5s + 4}$

Synthetically dividing the numerator by the denominator,

$$\mathrm{H}(s) = 3 - \frac{15s + 12}{s^2 + 5s + 4} = 3 - \left[\frac{16}{s+4} - \frac{1}{s+1}\right]$$

$$\mathrm{h}(t) = 3\delta(t) - \left[16e^{-4t} - e^{-t}\right]\mathrm{u}(t)$$

(b) $\mathrm{H}(s) = \dfrac{5s}{(s+1)^2}$

The partial-fraction expansion is

$$\mathrm{H}(s) = \frac{-5}{(s+1)^2} + \frac{5}{s+1}$$

$$\mathrm{h}(t) = 5e^{-t}(1-t)\mathrm{u}(t)$$

6. Find the inverse z transforms of these signals.

(a) $\mathrm{H}(z)=\dfrac{z^2}{z^2-1.1z+0.28}$

$$\frac{\mathrm{H}(z)}{z}=\frac{z}{z^2-1.1z+0.28}=\frac{7/3}{z-0.7}-\frac{4/3}{z-0.4}\Rightarrow \mathrm{H}(z)=\frac{7z/3}{z-0.7}-\frac{4z/3}{z-0.4}$$

$$\mathrm{h}[n]=\left[(7/3)(0.7)^n-(4/3)(0.4)^n\right]\mathrm{u}[n]$$

Alternate Solution:

$$\mathrm{H}(z)=1+\frac{1.1z-0.28}{z^2-1.1s+0.28}=1+\frac{1.633}{z-0.7}-\frac{0.533}{z-0.4}$$

$$\mathrm{h}[n]=\delta[n]+\left[1.633(0.7)^{n-1}-0.533(0.4)^{n-1}\right]\mathrm{u}[n-1]$$

(b) $\mathrm{H}(z)=\dfrac{z^2-2z+5}{z^2}$

$$\mathrm{H}(z)=\frac{z^2-2z+5}{z^2}=1-2z^{-1}+5z^{-2}$$

$$\mathrm{h}[n]=\delta[n]-2\delta[n-1]+5\delta[n-2]$$

7. Find the numerical Nyquist rates for these signals. (If a signal is not bandlimited write "infinity".)

(a) $\mathrm{x}(t)=-3\cos(200\pi t)\sin(500\pi t)$

$$\mathrm{X}(f)=(-3/2)\left[\delta(f-100)+\delta(f+100)\right]*(1/2)\left[\delta(f+250)-\delta(f-250)\right]$$

$$\mathrm{X}(f)=(-3/4)\left[\delta(f+150)-\delta(f-350)+\delta(f+350)-\delta(f-150)\right]$$

Highest frequency is 350 Hz. Therefore the Nyquist rate is 700 samples/second.

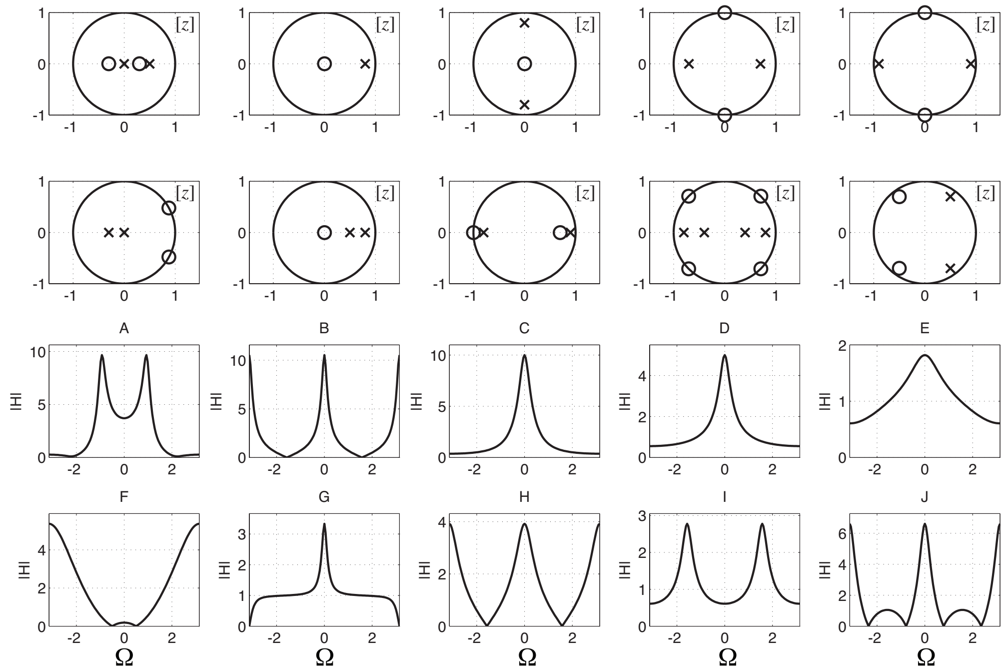
(b) $\mathrm{x}(t)=18\,\mathrm{sinc}(3t)*\delta_7(t)$

$$\mathrm{X}(f)=6\,\mathrm{rect}(f/3)\times(1/7)\delta_{1/7}(f)$$

The periodic impulse has impulses at every integer multiple of 1/7 Hz. The rectangle zeros out all impulses at frequencies greater than 1.5 in magnitude. Therefore the highest frequency is the greatest integer multiple of 1/7 that is less than 1.5. That is 10/7 or 1.429 and the Nyquist rate is 2.857 samples/second.

8. Match the magnitude frequency responses to the pole zero diagrams by writing the appropriate letter designation above the pole zero diagram.

E	D	I	H	B
F	C	G	J	A



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is approximated by a digital filter designed using the bilinear method.

(a) How many zeros will the digital filter have?

(b) Knowing only what this problem states, is it possible to locate the zeros numerically?

(c) If it is possible to locate the zeros numerically, where are they?

(d) How many poles will the digital filter have?

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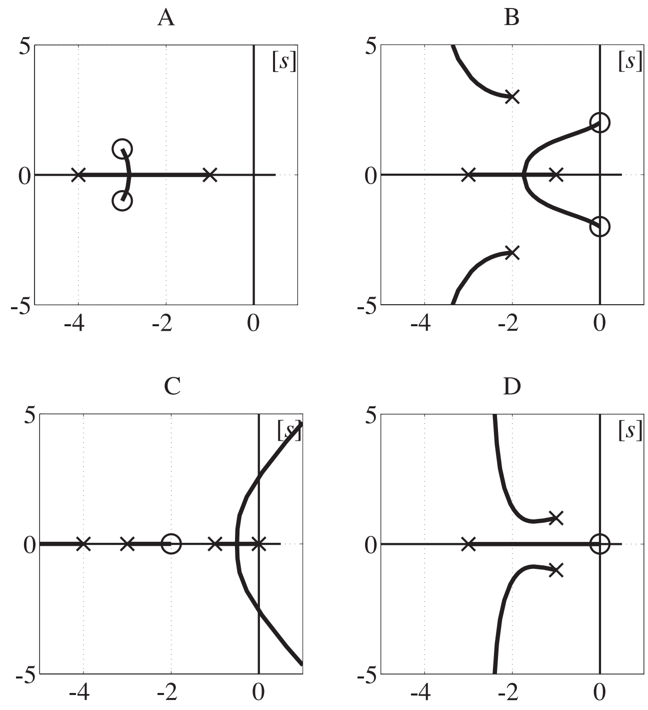
$$\mathrm{H}_a(s) = \frac{Ks}{s^2 + bs + c} \Rightarrow \mathrm{H}_d(z) = \frac{K\frac{2}{T_s}\frac{z-1}{z+1}}{\left(\frac{2}{T_s}\frac{z-1}{z+1}\right)^2 + b\frac{2}{T_s}\frac{z-1}{z+1} + c}$$

$$\mathrm{H}_d(z) = \frac{2KT_s(z+1)(z-1)}{4(z-1)^2 + 2bT_s(z+1)(z-1) + cT_s^2(z+1)^2}$$

Zeros at $z = \pm 1$. Two poles. Cannot locate the poles numerically with this information.

2. Below are pole/zero diagrams for the loop transfer functions of some feedback systems. Draw a root locus for each one.



3. A discrete time feedback system has a forward path transfer function $H_1(z) = \dfrac{Kz}{z-0.6}$ and a feedback path transfer function $H_2(z) = 1$. For what numerical range of *K*'s is this feedback system stable?

$$H(z) = \frac{\dfrac{Kz}{z-0.6}}{1+\dfrac{Kz}{z-0.6}} = \frac{Kz}{z(1+K)-0.6}$$

$$\text{Poles at } z(1+K)-0.6 = 0 \Rightarrow z = \frac{0.6}{1+K}$$

$$\text{For stability, } \left|\frac{0.6}{1+K}\right| < 1 \Rightarrow K > -0.4 \text{ or } K < -1.6$$

4. What are the numerical locations of the poles and zeros of a 2nd order butterworth highpass filter with a corner radian frequency of $\omega = 20$? (Specify the pole and zero locations in the form $A\angle B$ where A is the magnitude and B is the angle in radians. For example, $6\angle 1.4$.)

$$n = 2 \Rightarrow \text{Normalized lowpass poles at } e^{j3\pi/4}\ ,\ e^{-j3\pi/4} \Rightarrow \mathrm{H}_{LP}(s) = \frac{1}{\left(s - \dfrac{-1+j}{\sqrt{2}}\right)\left(s - \dfrac{-1-j}{\sqrt{2}}\right)}$$

$$\mathrm{H}_{LP}(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \Rightarrow \mathrm{H}_{HP}(s) = \left[\frac{1}{s^2 + \sqrt{2}s + 1}\right]_{s \to 20/s} = \frac{1}{(20/s)^2 + 20\sqrt{2}/s + 1} = \frac{s^2}{s^2 + 20\sqrt{2}s + 400}$$

$$\text{Two zeros at } s = 0 \text{ and Poles at } s = \frac{-20\sqrt{2} \pm \sqrt{800 - 1600}}{2} = -10\sqrt{2} \pm j\sqrt{200} = 20\angle 2.3562 \text{ and } 20\angle -2.3562$$

5. Find the inverse Laplace transforms of these signals.

(a) $\mathrm{H}(s) = \dfrac{7s^2}{s^2 + 5s + 4}$

Synthetically dividing the numerator by the denominator,

$$\mathrm{H}(s) = 7 - \frac{35s + 28}{s^2 + 5s + 4} = 7 - \left[\frac{112/3}{s+4} - \frac{7/3}{s+1}\right]$$

$$\mathrm{h}(t) = 7\delta(t) - \left[37.33e^{-4t} - 2.33e^{-t}\right]\mathrm{u}(t)$$

(b) $\mathrm{H}(s) = \dfrac{5s}{(s+2)^2}$

The partial-fraction expansion is

$$\mathrm{H}(s) = \frac{-10}{(s+2)^2} + \frac{5}{s+2}$$

$$\mathrm{h}(t) = 5e^{-2t}(1 - 2t)\mathrm{u}(t)$$

6. Find the inverse z transforms of these signals.

(a) $\mathrm{H}(z)=\dfrac{z^2}{z^2-0.7z+0.12}$

$$\frac{\mathrm{H}(z)}{z}=\frac{z}{z^2-0.7z+0.12}=-\frac{3}{z-0.3}+\frac{4}{z-0.4}\Rightarrow \mathrm{H}(z)=-\frac{3z}{z-0.3}+\frac{4z}{z-0.4}$$

$$\mathrm{h}[n]=\left[4(0.4)^n-3(0.3)^n\right]\mathrm{u}[n]$$

Alternate Solution:

$$\mathrm{H}(z)=1+\frac{0.7z-0.12}{z^2-0.7z+0.12}=1+\frac{1.6}{z-0.4}-\frac{0.9}{z-0.3}$$

$$\mathrm{h}[n]=\delta[n]+\left[1.6(0.4)^{n-1}-0.9(0.3)^{n-1}\right]\mathrm{u}[n-1]$$

(b) $\mathrm{H}(z)=\dfrac{3z^2-z+4}{z^2}$

$$\mathrm{H}(z)=\frac{3z^2-z+4}{z^2}=3-z^{-1}+4z^{-2}$$

$$\mathrm{h}[n]=3\delta[n]-\delta[n-1]+4\delta[n-2]$$

7. Find the numerical Nyquist rates for these signals. (If a signal is not bandlimited write "infinity".)

(a) $\mathrm{x}(t)=-3\cos(200\pi t)\sin(1000\pi t)$

$$\mathrm{X}(f)=(-3/2)\left[\delta(f-100)+\delta(f+100)\right]*(1/2)\left[\delta(f+500)-\delta(f-500)\right]$$

$$\mathrm{X}(f)=(-3/4)\left[\delta(f+400)-\delta(f-600)+\delta(f+600)-\delta(f-400)\right]$$

Highest frequency is 600 Hz. Therefore the Nyquist rate is 1200 samples/second.

(b) $\mathrm{x}(t)=18\,\mathrm{sinc}(3t)*\delta_9(t)$

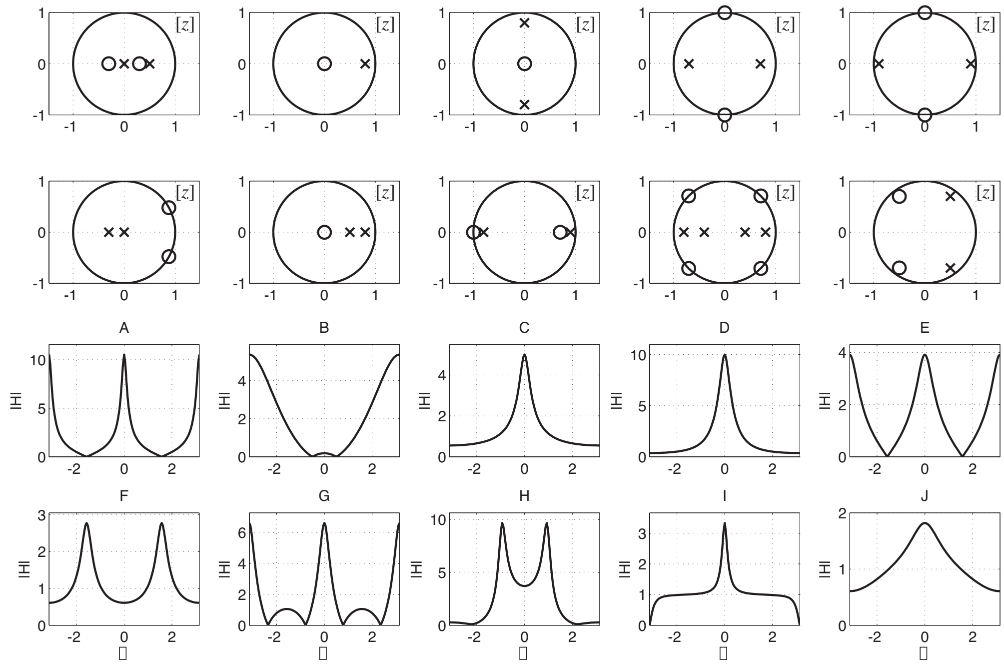
$$\mathrm{X}(f)=6\,\mathrm{rect}(f/3)\times(1/9)\delta_{1/9}(f)$$

The periodic impulse has impulses at every integer multiple of 1/9 Hz. The rectangle zeros out all impulses at frequencies greater than 1.5 in magnitude. Therefore the highest frequency is the greatest integer multiple of 1/9 that is less than 1.5. That is 13/9 or 1.444 and the Nyquist rate is 2.888 samples/second.

8. Match the magnitude frequency responses to the pole zero diagrams by writing the appropriate letter designation above the pole zero diagram.

J C F E A

B D I G H



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$$\mathrm{H}_d(z) = \frac{2KT_s(z+1)(z-1)}{4(z-1)^2 + 2bT_s(z+1)(z-1) + cT_s^2(z+1)^2}$$

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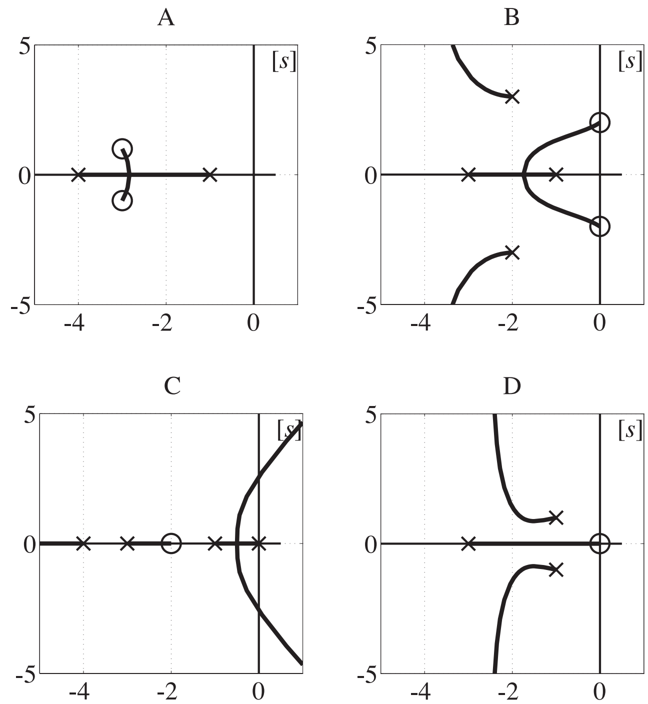

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$$\mathrm{H}(z) = \frac{\dfrac{Kz}{z+0.1}}{1+\dfrac{Kz}{z+0.1}} = \frac{Kz}{z(1+K)+0.1}$$

$$\text{Poles at } z(1+K)+0.1 = 0 \Rightarrow z = -\frac{0.1}{1+K}$$

$$\text{For stability, } \left|-\frac{0.1}{1+K}\right| < 1 \Rightarrow K > -0.9 \text{ or } K < -1.1$$

4. What are the numerical locations of the poles and zeros of a 2nd order butterworth highpass filter with a corner radian frequency of $\omega = 30$? (Specify the pole and zero locations in the form $A\angle B$ where A is the magnitude and B is the angle in radians. For example, $6\angle 1.4$.)

$$n = 2 \Rightarrow \text{Normalized lowpass poles at } e^{j3\pi/4}\ ,\ e^{-j3\pi/4} \Rightarrow \mathrm{H}_{LP}(s) = \frac{1}{\left(s - \frac{-1+j}{\sqrt{2}}\right)\left(s - \frac{-1-j}{\sqrt{2}}\right)}$$

$$\mathrm{H}_{LP}(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \Rightarrow \mathrm{H}_{HP}(s) = \left[\frac{1}{s^2 + \sqrt{2}s + 1}\right]_{s \to 30/s} = \frac{1}{(30/s)^2 + 30\sqrt{2}/s + 1} = \frac{s^2}{s^2 + 30\sqrt{2}s + 900}$$

$$\text{Two zeros at } s = 0 \text{ and Poles at } s = \frac{-30\sqrt{2} \pm \sqrt{1800 - 3600}}{2} = -15\sqrt{2} \pm j\sqrt{450} = 30\angle 2.3562 \text{ and } 30\angle -2.3562$$

5. Find the inverse Laplace transforms of these signals.

(a) $\mathrm{H}(s) = \dfrac{5s^2}{s^2 + 5s + 4}$

Synthetically dividing the numerator by the denominator,

$$\mathrm{H}(s) = 5 - \frac{25s + 20}{s^2 + 5s + 4} = 5 - \left[\frac{80/3}{s+4} - \frac{5/3}{s+1}\right]$$

$$\mathrm{h}(t) = 5\delta(t) - \left[26.667e^{-4t} - 1.667e^{-t}\right]\mathrm{u}(t)$$

(b) $\mathrm{H}(s) = \dfrac{5s}{(s+3)^2}$

The partial-fraction expansion is

$$\mathrm{H}(s) = \frac{-15}{(s+3)^2} + \frac{5}{s+3}$$

$$\mathrm{h}(t) = 5e^{-3t}(1 - 3t)\mathrm{u}(t)$$

6. Find the inverse z transforms of these signals.

(a) $\mathrm{H}(z) = \dfrac{z^2}{z^2 - 0.7z + 0.1}$

$$\frac{\mathrm{H}(z)}{z} = \frac{z}{z^2 - 0.7z + 0.1} = \frac{5/3}{z-0.5} - \frac{2/3}{z-0.2} \Rightarrow \mathrm{H}(z) = \frac{5z/3}{z-0.5} - \frac{2z/3}{z-0.2}$$

$$\mathrm{h}[n] = \left[(5/3)(0.5)^n - (2/3)(0.2)^n\right]\mathrm{u}[n]$$

Alternate Solution:

$$\mathrm{H}(z) = 1 + \frac{0.7z - 0.1}{z^2 - 0.7z + 0.1} = 1 + \frac{0.833}{z-0.5} - \frac{0.133}{z-0.2}$$

$$\mathrm{h}[n] = \delta[n] + \left[0.833(0.5)^{n-1} - 0.133(0.2)^{n-1}\right]\mathrm{u}[n-1]$$

(b) $\mathrm{H}(z) = \dfrac{3z^2 - 5z + 1}{z^2}$

$$\mathrm{H}(z) = \frac{3z^2 - 5z + 1}{z^2} = 3 - 5z^{-1} + z^{-2}$$

$$\mathrm{h}[n] = 3\delta[n] - 5\delta[n-1] + \delta[n-2]$$

7. Find the numerical Nyquist rates for these signals. (If a signal is not bandlimited write "infinity".)

(a) $\mathrm{x}(t) = -3\cos(260\pi t)\sin(500\pi t)$

$$\mathrm{X}(f) = (-3/2)\left[\delta(f-130) + \delta(f+130)\right] * (1/2)\left[\delta(f+250) - \delta(f-250)\right]$$

$$\mathrm{X}(f) = (-3/4)\left[\delta(f+120) - \delta(f-380) + \delta(f+380) - \delta(f-120)\right]$$

Highest frequency is 380 Hz. Therefore the Nyquist rate is 760 samples/second.

(b) $\mathrm{x}(t) = 18\,\mathrm{sinc}(3t) * \delta_{11}(t)$

$$\mathrm{X}(f) = 6\,\mathrm{rect}(f/3) \times (1/11)\delta_{1/11}(f)$$

The periodic impulse has impulses at every integer multiple of 1/11 Hz. The rectangle zeros out all impulses at frequencies greater than 1.5 in magnitude. Therefore the highest frequency is the greatest integer multiple of 1/1 that is less than 1.5. That is 16/11 or 1.455 and the Nyquist rate is 2.91 samples/second.

8. Match the magnitude frequency responses to the pole zero diagrams by writing the appropriate letter designation above the pole zero diagram.

F	D	J	I	H
B	A	C	E	G