# Solution of ECE 316 Final Examination S10

1. The transfer function for the system below can be written in the form

$$\mathrm{H}(s)=\frac{b_2 s^2+b_1 s+b_0}{s^2+a_1 s+a_0}$$

(a) Find numerical values for the $a$'s and $b$'s.

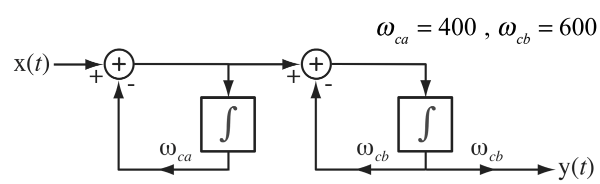
$$\mathrm{H}(s)=\frac{s}{s+\omega_{ca}}\times\frac{\omega_{cb}}{s+\omega_{cb}}=\frac{\omega_{cb}s}{s^2+(\omega_{ca}+\omega_{cb})s+\omega_{ca}\omega_{cb}}=\frac{600s}{s^2+1000s+240{,}000}$$

$b_2=0$ , $b_1=600$ , $b_0=0$ , $a_1=1000$ , $a_0=240{,}000$

(b) Find the system's numerical frequency response magnitude in dB and phase in radians at 150 Hz.

$$\mathrm{H}(j2\pi\times150)=\frac{j600\times300\pi}{(j300\pi)^2+1000\times j300\pi+240{,}000}=0.49435\angle-0.60251\text{ radians}$$

$$\left|\mathrm{H}(j2\pi\times150)\right|_{\mathrm{dB}}=20\log10(0.49435)=-6.1193\text{ dB}$$



2. A continuous-time signal of the form

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

is impulse-sampled at a rate of $f_s$ samples/second, then lowpass filtered by an ideal analog lowpass filter whose frequency response is $H(f) = \text{rect}(f / 2f_c)$ to produce the response $y(t)$. If $y(t) \xleftrightarrow{\mathcal{F}} Y(f)$, for each case below, list the frequencies of all the impulses in $Y(f)$ in the range $0 \le f < \infty$.

(a) $f_1 = 200$ Hz , $f_2 = 250$ Hz , $f_s = 1000$ Hz , $f_c = 900$ Hz

Frequencies of impulses in $Y(f)$ 200, 250, 750, 800 (Hz)

(b) $f_1 = 200$ Hz , $f_2 = 250$ Hz , $f_s = 450$ Hz , $f_c = 900$ Hz

Frequencies of impulses in $Y(f)$ 200, 250, 650, 700 (Hz)

(c) $f_1 = 200$ Hz , $f_2 = 250$ Hz , $f_s = 320$ Hz , $f_c = 500$ Hz

Frequencies of impulses in $Y(f)$ 70, 120, 200, 250, 390, 440 (Hz)

(d) $f_1 = 200$ Hz , $f_2 = 250$ Hz , $f_s = 150$ Hz , $f_c = 450$ Hz

Frequencies of impulses in $Y(f)$ 50, 100, 200, 250, 350, 400 (Hz)

3. In the DSBTC modulator below, let $x(t) = \text{rect}(t)$.

(a) Find the numerical value of $y(10\text{ ms})$.

$$y(t) = (2 + 0.8\,\text{rect}(t))\cos(40\pi t) \Rightarrow y(10\text{ ms}) = (2 + 0.8\,\text{rect}(0.01))\cos(0.40\pi) = 0.8652$$

(b) If $y(t) \xleftrightarrow{\mathcal{F}} Y(f)$, find the numerical magnitude and phase (in radians) of $Y(21.5\text{ Hz})$.

$$Y(f) = [2\delta(f) + 0.8\,\text{sinc}(f)] * (1/2)[\delta(f-20) + \delta(f+20)]$$

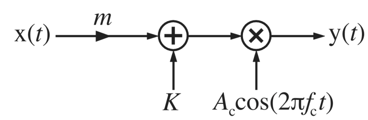
$$Y(f) = (1/2)\{2\delta(f) * [\delta(f-20) + \delta(f+20)] + 0.8\,\text{sinc}(f) * [\delta(f-20) + \delta(f+20)]\}$$

$$Y(f) = (1/2)\{2[\delta(f-20) + \delta(f+20)] + 0.8[\text{sinc}(f-20) + \text{sinc}(f+20)]\}$$

$$Y(21.5\text{ Hz}) = (1/2)\{2[\delta(21.5-20) + \delta(21.5+20)] + 0.8[\text{sinc}(21.5-20) + \text{sinc}(21.5+20)]\}$$

$$Y(21.5\text{ Hz}) = 0.4[\text{sinc}(1.5) + \text{sinc}(41.5)] = -0.088 = 0.088\angle \pm\pi$$

$m = 0.8$, $K = 2$, $A_c = 1$, $f_c = 20$



4. An analog filter has a transfer function $\mathrm{H}_a(s) = \dfrac{s^2 + 2500}{s(s+100)}$.

(a) Find the numerical zero and pole locations for the analog filter.

$$s^2 + 2500 = 0 \Rightarrow \text{Zeros at } s = \pm j50$$
$$s(s+100) = 0 \Rightarrow \text{Poles at } s = 0, -100$$

(b) Find the numerical magnitude and phase of the frequency response of the analog filter at $f = 40$ Hz and $f \to \infty$.

$$\mathrm{H}_a(j2\pi \times 40) = \frac{-(80\pi)^2 + 2500}{j80\pi(j80\pi + 100)} = 0.8924\angle 0.3787$$

$$\mathrm{H}_a(+j\infty) = 1\angle 0$$

(c) Using a sampling rate of 1000 Hz and the bilinear method, find the transfer function $\mathrm{H}_d(z)$ of a digital filter designed to approximate it.

$$\mathrm{H}_d(z) = \mathrm{H}_a(s)_{s \to \frac{2}{T_s}\frac{z-1}{z+1}} = \frac{\left(2000\dfrac{z-1}{z+1}\right)^2 + 2500}{\left(2000\dfrac{z-1}{z+1}\right)\left(2000\dfrac{z-1}{z+1} + 100\right)} = 0.953\frac{z^2 - 1.9975z + 1}{z^2 - 1.90476z + 0.90476}$$

(d) What are the numerical magnitudes and phases of the zeros and poles of the digital filter? (The configuration of zeros and poles for the digital filter should look similar to the configuration of zeros and poles for the analog filter with the portion of the unit circle near $z$=1 corresponding to the $\omega$ axis in the $s$ plane.)

$$z^2 - 1.9975z + 1 = 0 \Rightarrow \text{Zeros at } z = 1\angle \pm 0.05$$
$$z^2 - 1.9048z + 0.9048 = 0 \Rightarrow \text{Poles at } z = 0.90476\angle 0\ ,\ 1\angle 0$$

(e) What are the magnitude and phase of the digital filter frequency response at $\Omega = 0.08\pi$ and $\Omega = \pi$? What is the relationship between these digital filter frequency responses and those of the analog filter in part (b) above?

$$\mathrm{H}_d\left(e^{j0.08\pi}\right) = \mathrm{H}(1\angle 0.25133) = 0.953\frac{-1 - j1.9975 + 1}{-1 - j1.90476 + 0.90476} = 0.89341\angle 0.37687$$

$$\mathrm{H}_d\left(e^{j\pi}\right) = \mathrm{H}(-1) = 0.953\frac{1 + 1.9975 + 1}{1 + 1.90476 + 0.90476} = 1\angle 0$$

# Solution of ECE 316 Final Examination S10

1. The transfer function for the system below can be written in the form

$$\mathrm{H}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

(a) Find numerical values for the $a$'s and $b$'s.

$$\mathrm{H}(s) = \frac{s}{s+\omega_{ca}} \times \frac{\omega_{cb}}{s+\omega_{cb}} = \frac{\omega_{cb} s}{s^2 + (\omega_{ca}+\omega_{cb})s + \omega_{ca}\omega_{cb}} = \frac{800s}{s^2 + 1200s + 320{,}000}$$

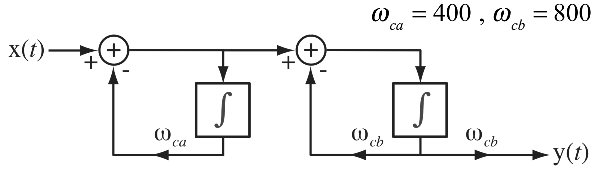
$b_2 = 0$ , $b_1 = 800$ , $b_0 = 0$ , $a_1 = 1200$ , $a_0 = 320{,}000$

(b) Find the system's numerical frequency response magnitude in dB and phase in radians at 150 Hz.

$$\mathrm{H}(j2\pi \times 150) = \frac{j800 \times 300\pi}{(j300\pi)^2 + 1200 \times j300\pi + 320{,}000} = 0.5957\angle -0.46561 \text{ radians}$$

$$\left|\mathrm{H}(j2\pi \times 150)\right|_{\mathrm{dB}} = 20\log10(0.5957) = -4.499 \text{ dB}$$

$\omega_{ca} = 400$ , $\omega_{cb} = 800$



2. A continuous-time signal of the form

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

is impulse-sampled at a rate of $f_s$ samples/second, then lowpass filtered by an ideal analog lowpass filter whose frequency response is $H(f) = \text{rect}(f / 2f_c)$ to produce the response $y(t)$. If $y(t) \xleftrightarrow{\mathcal{F}} Y(f)$, for each case below, list the frequencies of all the impulses in $Y(f)$ in the range $0 \le f < \infty$.

(a) $f_1 = 400$ Hz , $f_2 = 500$ Hz , $f_s = 2000$ Hz , $f_c = 1800$ Hz

Frequencies of impulses in $Y(f)$ 400, 500, 1500, 1600 (Hz)

(b) $f_1 = 400$ Hz , $f_2 = 500$ Hz , $f_s = 900$ Hz , $f_c = 1800$ Hz

Frequencies of impulses in $Y(f)$ 400, 500, 1300, 1400 (Hz)

(c) $f_1 = 400$ Hz , $f_2 = 500$ Hz , $f_s = 640$ Hz , $f_c = 1000$ Hz

Frequencies of impulses in $Y(f)$ 140, 240, 400, 500, 780, 880 (Hz)

(d) $f_1 = 400$ Hz , $f_2 = 500$ Hz , $f_s = 300$ Hz , $f_c = 900$ Hz

Frequencies of impulses in $Y(f)$ 100, 200, 400, 500, 700, 800 (Hz)

3. In the DSBTC modulator below, let $\mathrm{x}(t) = \mathrm{rect}(t)$.

(a) Find the numerical value of $\mathrm{y}(10\text{ ms})$.

$$\mathrm{y}(t) = (1.5 + 0.6\,\mathrm{rect}(t))\cos(40\pi t) \Rightarrow \mathrm{y}(10\text{ ms}) = (1.5 + 0.6\,\mathrm{rect}(0.01))\cos(0.40\pi) = 0.6489$$

(b) If $\mathrm{y}(t) \xleftrightarrow{\mathcal{F}} \mathrm{Y}(f)$, find the numerical magnitude and phase (in radians) of $\mathrm{Y}(21.5\text{ Hz})$.

$$\mathrm{Y}(f) = [1.5\delta(f) + 0.6\,\mathrm{sinc}(f)] * (1/2)[\delta(f-20) + \delta(f+20)]$$

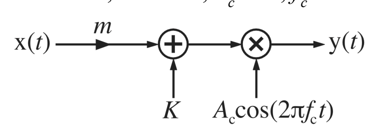
$$\mathrm{Y}(f) = (1/2)\{1.5\delta(f) * [\delta(f-20) + \delta(f+20)] + 0.6\,\mathrm{sinc}(f) * [\delta(f-20) + \delta(f+20)]\}$$

$$\mathrm{Y}(f) = (1/2)\{1.5[\delta(f-20) + \delta(f+20)] + 0.6[\mathrm{sinc}(f-20) + \mathrm{sinc}(f+20)]\}$$

$$\mathrm{Y}(21.5\text{ Hz}) = (1/2)\{1.5[\delta(21.5-20) + \delta(21.5+20)] + 0.6[\mathrm{sinc}(21.5-20) + \mathrm{sinc}(21.5+20)]\}$$

$$\mathrm{Y}(21.5\text{ Hz}) = 0.3[\mathrm{sinc}(1.5) + \mathrm{sinc}(41.5)] = -0.066 = 0.066\angle \pm \pi$$

$m = 0.6$, $K = 1.5$, $A_c = 1$, $f_c = 20$



4. An analog filter has a transfer function $\mathrm{H}_a(s) = \dfrac{s^2 + 4900}{s(s+200)}$.

(a) Find the numerical zero and pole locations for the analog filter.

$$s^2 + 4900 = 0 \Rightarrow \text{Zeros at } s = \pm j70$$
$$s(s+200) = 0 \Rightarrow \text{Poles at } s = 0, -200$$

(b) Find the numerical magnitude and phase of the frequency response of the analog filter at $f = 40$ Hz and $f \to \infty$.

$$\mathrm{H}_a(j2\pi \times 40) = \frac{-(80\pi)^2 + 4900}{j80\pi(j80\pi + 200)} = 0.72178\angle 0.67216$$

$$\mathrm{H}_a(+j\infty) = 1\angle 0$$

(c) Using a sampling rate of 1000 Hz and the bilinear method, find the transfer function $\mathrm{H}_d(z)$ of a digital filter designed to approximate it.

$$\mathrm{H}_d(z) = \mathrm{H}_a(s)\Big|_{s \to \frac{2}{T_s}\frac{z-1}{z+1}} = \frac{\left(2000\frac{z-1}{z+1}\right)^2 + 4900}{\left(2000\frac{z-1}{z+1}\right)\left(2000\frac{z-1}{z+1} + 200\right)} = 0.9102\frac{z^2 - 1.9951z + 1}{z^2 - 1.8182z + 0.818}$$

(d) What are the numerical magnitudes and phases of the zeros and poles of the digital filter?
(The configuration of zeros and poles for the digital filter should look similar to the configuration of zeros and poles for the analog filter with the portion of the unit circle near $z$=1 corresponding to the $\omega$ axis in the $s$ plane.)

$$z^2 - 1.9951z + 1 = 0 \Rightarrow \text{Zeros at } z = 1\angle \pm 0.07$$
$$z^2 - 1.8182z + 0.8182 = 0 \Rightarrow \text{Poles at } z = 0.8182\angle 0\ ,\ 1\angle 0$$

(e) What are the magnitude and phase of the digital filter frequency response at $\Omega = 0.08\pi$ and $\Omega = \pi$? What is the relationship between these digital filter frequency responses and those of the analog filter in part (b) above?

$$\mathrm{H}_d\left(e^{j0.08\pi}\right) = \mathrm{H}(1\angle 0.25133) = 0.9102\frac{(1\angle 0.25133)^2 - 1.9951(1\angle 0.25133) + 1}{(1\angle 0.25133)^2 - 1.8182(1\angle 0.25133) + 0.8182} = 0.72389\angle 0.6696$$

$$\mathrm{H}_d\left(e^{j\pi}\right) = \mathrm{H}(-1) = 0.953\frac{1 + 1.9975 + 1}{1 + 1.8182 + 0.8182} = 1\angle 0$$

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# Solution of ECE 316 Final Examination S10

1. The transfer function for the system below can be written in the form

$$\mathrm{H}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

(a) Find numerical values for the $a$'s and $b$'s.

$$\mathrm{H}(s) = \frac{s}{s+\omega_{ca}} \times \frac{\omega_{cb}}{s+\omega_{cb}} = \frac{\omega_{cb} s}{s^2 + (\omega_{ca}+\omega_{cb})s + \omega_{ca}\omega_{cb}} = \frac{700s}{s^2 + 1100s + 280{,}000}$$

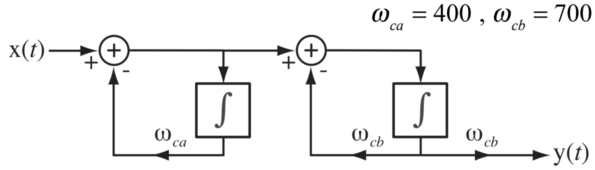
$b_2 = 0$ , $b_1 = 700$ , $b_0 = 0$ , $a_1 = 1100$ , $a_0 = 280{,}000$

(b) Find the system's numerical frequency response magnitude in dB and phase in radians at 150 Hz.

$$\mathrm{H}(j2\pi \times 150) = \frac{j700 \times 300\pi}{(j300\pi)^2 + 1100 \times j300\pi + 280{,}000} = 0.54887\angle -0.5306 \text{ radians}$$

$$\left|\mathrm{H}(j2\pi \times 150)\right|_{\mathrm{dB}} = 20\log10(0.54887) = -5.2106 \text{ dB}$$

$\omega_{ca} = 400$ , $\omega_{cb} = 700$



2. A continuous-time signal of the form

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

is impulse-sampled at a rate of $f_s$ samples/second, then lowpass filtered by an ideal analog lowpass filter whose frequency response is $H(f) = \mathrm{rect}(f/2f_c)$ to produce the response $y(t)$. If $y(t) \xleftrightarrow{\mathcal{F}} Y(f)$, for each case below, list the frequencies of all the impulses in $Y(f)$ in the range $0 \le f < \infty$.

(a) $f_1 = 100$ Hz , $f_2 = 125$ Hz , $f_s = 500$ Hz , $f_c = 450$ Hz

Frequencies of impulses in $Y(f)$ 100, 125, 375, 400 (Hz)

(b) $f_1 = 100$ Hz , $f_2 = 125$ Hz , $f_s = 225$ Hz , $f_c = 450$ Hz

Frequencies of impulses in $Y(f)$ 100, 125, 325, 350 (Hz)

(c) $f_1 = 100$ Hz , $f_2 = 125$ Hz , $f_s = 160$ Hz , $f_c = 250$ Hz

Frequencies of impulses in $Y(f)$ 35, 60, 100, 125, 195, 220 (Hz)

(d) $f_1 = 100$ Hz , $f_2 = 125$ Hz , $f_s = 75$ Hz , $f_c = 250$ Hz

Frequencies of impulses in $Y(f)$ 25, 50, 100, 125, 200, 225, 250 (half-size) (Hz)

3. In the DSBTC modulator below, let $\mathrm{x}(t)=\mathrm{rect}(t)$.

(a) Find the numerical value of $\mathrm{y}(10\text{ ms})$.

$$\mathrm{y}(t)=(2.5+0.9\,\mathrm{rect}(t))\cos(40\pi t)\Rightarrow \mathrm{y}(10\text{ ms})=(2.5+0.9\,\mathrm{rect}(0.01))\cos(0.40\pi)=1.0507$$

(b) If $\mathrm{y}(t)\xleftrightarrow{\mathcal{F}}\mathrm{Y}(f)$, find the numerical magnitude and phase (in radians) of $\mathrm{Y}(21.5\text{ Hz})$.

$$\mathrm{Y}(f)=\left[2.5\delta(f)+0.9\,\mathrm{sinc}(f)\right]*(1/2)\left[\delta(f-20)+\delta(f+20)\right]$$

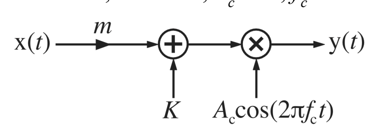
$$\mathrm{Y}(f)=(1/2)\left\{2.5\delta(f)*\left[\delta(f-20)+\delta(f+20)\right]+0.9\,\mathrm{sinc}(f)*\left[\delta(f-20)+\delta(f+20)\right]\right\}$$

$$\mathrm{Y}(f)=(1/2)\left\{2.5\left[\delta(f-20)+\delta(f+20)\right]+0.9\left[\mathrm{sinc}(f-20)+\mathrm{sinc}(f+20)\right]\right\}$$

$$\mathrm{Y}(21.5\text{ Hz})=(1/2)\left\{2.5\left[\delta(21.5-20)+\delta(21.5+20)\right]+0.9\left[\mathrm{sinc}(21.5-20)+\mathrm{sinc}(21.5+20)\right]\right\}$$

$$\mathrm{Y}(21.5\text{ Hz})=0.45\left[\mathrm{sinc}(1.5)+\mathrm{sinc}(41.5)\right]=-0.0686=0.0989\angle\pm\pi$$

$m=0.9$ , $K=2.5$ , $A_c=1$ , $f_c=20$



4. An analog filter has a transfer function $\mathrm{H}_a(s) = \dfrac{s^2 + 1600}{s(s+50)}$.

(a) Find the numerical zero and pole locations for the analog filter.

$$s^2 + 1600 = 0 \Rightarrow \text{Zeros at } s = \pm j40$$
$$s(s+50) = 0 \Rightarrow \text{Poles at } s = 0, -50$$

(b) Find the numerical magnitude and phase of the frequency response of the analog filter at $f = 40$ Hz and $f \to \infty$.

$$\mathrm{H}_a(j2\pi \times 40) = \frac{-(80\pi)^2 + 1600}{j80\pi(j80\pi + 50)} = 0.95594\angle 0.19638$$

$$\mathrm{H}_a(+j\infty) = 1\angle 0$$

(c) Using a sampling rate of 1000 Hz and the bilinear method, find the transfer function $\mathrm{H}_d(z)$ of a digital filter designed to approximate it.

$$\mathrm{H}_d(z) = \mathrm{H}_a(s)_{s \to \frac{2}{T_s}\frac{z-1}{z+1}} = \frac{\left(2000\frac{z-1}{z+1}\right)^2 + 2500}{\left(2000\frac{z-1}{z+1}\right)\left(2000\frac{z-1}{z+1} + 100\right)} = 0.976\frac{z^2 - 1.9984z + 1}{z^2 - 1.95122z + 0.95122}$$

(d) What are the numerical magnitudes and phases of the zeros and poles of the digital filter? (The configuration of zeros and poles for the digital filter should look similar to the configuration of zeros and poles for the analog filter with the portion of the unit circle near $z$=1 corresponding to the $\omega$ axis in the $s$ plane.)

$$z^2 - 1.9984z + 1 = 0 \Rightarrow \text{Zeros at } z = 1\angle \pm 0.04$$
$$z^2 - 1.95122z + 0.95122 = 0 \Rightarrow \text{Poles at } z = 0.95122\angle 0 \text{ , } 1\angle 0$$

(e) What are the magnitude and phase of the digital filter frequency response at $\Omega = 0.08\pi$ and $\Omega = \pi$? What is the relationship between these digital filter frequency responses and those of the analog filter in part (b) above?

$$\mathrm{H}_d\left(e^{j0.08\pi}\right) = \mathrm{H}(1\angle 0.25133) = 0.953\frac{-1 - j1.9984 + 1}{-1 - j1.95122 + 0.95122} = 0.95639\angle 0.19537$$

$$\mathrm{H}_d\left(e^{j\pi}\right) = \mathrm{H}(-1) = 0.953\frac{1 + 1.9984 + 1}{1 + 1.95122 + 0.95122} = 1\angle 0$$

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1. The transfer function for the system below can be written in the form

$$\mathrm{H}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

(a) Find numerical values for the $a$'s and $b$'s.

$$\mathrm{H}(s) = \frac{s}{s+\omega_{ca}} \times \frac{\omega_{cb}}{s+\omega_{cb}} = \frac{\omega_{cb} s}{s^2 + (\omega_{ca}+\omega_{cb})s + \omega_{ca}\omega_{cb}} = \frac{900s}{s^2 + 1300s + 360{,}000}$$

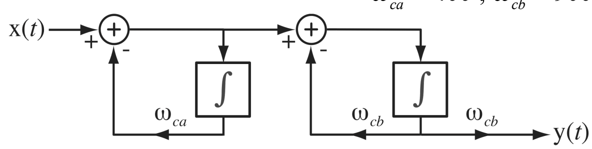
$b_2 = 0$ , $b_1 = 900$ , $b_0 = 0$ , $a_1 = 1300$ , $a_0 = 360{,}000$

(b) Find the system's numerical frequency response magnitude in dB and phase in radians at 150 Hz.

$$\mathrm{H}(j2\pi \times 150) = \frac{j900 \times 300\pi}{(j300\pi)^2 + 1300 \times j300\pi + 360{,}000} = 0.63573\angle -0.40708 \text{ radians}$$

$$\left|\mathrm{H}(j2\pi \times 150)\right|_{\mathrm{dB}} = 20\log10(0.63573) = -3.9345 \text{ dB}$$

$\omega_{ca} = 400$ , $\omega_{cb} = 900$



2. A continuous-time signal of the form

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

is impulse-sampled at a rate of $f_s$ samples/second, then lowpass filtered by an ideal analog lowpass filter whose frequency response is $H(f) = \text{rect}(f / 2f_c)$ to produce the response $y(t)$. If $y(t) \xleftrightarrow{\mathcal{F}} Y(f)$, for each case below, list the frequencies of all the impulses in $Y(f)$ in the range $0 \le f < \infty$.

(a) $f_1 = 800$ Hz , $f_2 = 1000$ Hz , $f_s = 4000$ Hz , $f_c = 3600$ Hz

Frequencies of impulses in $Y(f)$ 800, 1000, 3000, 3200 (Hz)

(b) $f_1 = 800$ Hz , $f_2 = 1000$ Hz , $f_s = 1800$ Hz , $f_c = 3600$ Hz

Frequencies of impulses in $Y(f)$ 800, 1000, 2600, 2800 (Hz)

(c) $f_1 = 800$ Hz , $f_2 = 1000$ Hz , $f_s = 1280$ Hz , $f_c = 2000$ Hz

Frequencies of impulses in $Y(f)$ 280, 480, 800, 1000, 1560, 1760 (Hz)

(d) $f_1 = 800$ Hz , $f_2 = 1000$ Hz , $f_s = 600$ Hz , $f_c = 1800$ Hz

Frequencies of impulses in $Y(f)$ 200, 400, 800, 1000, 1400, 1600 (Hz)

3. In the DSBTC modulator below, let $\text{x}(t) = \text{rect}(t)$.

(a) Find the numerical value of $\text{y}(10\text{ ms})$.

$$\text{y}(t) = \left(3 + 0.5\,\text{rect}(t)\right)\cos(40\pi t) \Rightarrow \text{y}(10\text{ ms}) = \left(3 + 0.5\,\text{rect}(0.01)\right)\cos(0.40\pi) = 1.0816$$

(b) If $\text{y}(t) \xleftrightarrow{\mathcal{F}} \text{Y}(f)$, find the numerical magnitude and phase (in radians) of $\text{Y}(21.5\text{ Hz})$.

$$\text{Y}(f) = \left[3\delta(f) + 0.5\,\text{sinc}(f)\right] * (1/2)\left[\delta(f-20) + \delta(f+20)\right]$$

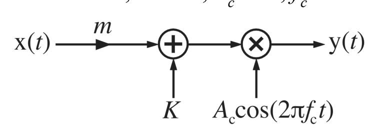
$$\text{Y}(f) = (1/2)\left\{3\delta(f) * \left[\delta(f-20) + \delta(f+20)\right] + 0.5\,\text{sinc}(f) * \left[\delta(f-20) + \delta(f+20)\right]\right\}$$

$$\text{Y}(f) = (1/2)\left\{3\left[\delta(f-20) + \delta(f+20)\right] + 0.5\left[\text{sinc}(f-20) + \text{sinc}(f+20)\right]\right\}$$

$$\text{Y}(21.5\text{ Hz}) = (1/2)\left\{3\left[\delta(21.5-20) + \delta(21.5+20)\right] + 0.5\left[\text{sinc}(21.5-20) + \text{sinc}(21.5+20)\right]\right\}$$

$$\text{Y}(21.5\text{ Hz}) = 0.25\left[\text{sinc}(1.5) + \text{sinc}(41.5)\right] = -0.055 = 0.055\angle \pm \pi$$

$m = 0.5$, $K = 3$, $A_c = 1$, $f_c = 20$



4. An analog filter has a transfer function $\mathrm{H}_a(s) = \dfrac{s^2 + 900}{s(s+400)}$.

(a) Find the numerical zero and pole locations for the analog filter.

$$s^2 + 900 = 0 \Rightarrow \text{Zeros at } s = \pm j30$$
$$s(s+400) = 0 \Rightarrow \text{Poles at } s = 0, -400$$

(b) Find the numerical magnitude and phase of the frequency response of the analog filter at $f = 40$ Hz and $f \to \infty$.

$$\mathrm{H}_a(j2\pi \times 40) = \frac{-(80\pi)^2 + 900}{j80\pi(j80\pi + 400)} = 0.52444\angle 1.0098$$

$$\mathrm{H}_a(+j\infty) = 1\angle 0$$

(c) Using a sampling rate of 1000 Hz and the bilinear method, find the transfer function $\mathrm{H}_d(z)$ of a digital filter designed to approximate it.

$$\mathrm{H}_d(z) = \mathrm{H}_a(s)_{s \to \frac{2}{T_s}\frac{z-1}{z+1}} = \frac{\left(2000\dfrac{z-1}{z+1}\right)^2 + 2500}{\left(2000\dfrac{z-1}{z+1}\right)\left(2000\dfrac{z-1}{z+1} + 100\right)} = 0.8335\frac{z^2 - 1.9991z + 1}{z^2 - 1.6667z + 0.6667}$$

(d) What are the numerical magnitudes and phases of the zeros and poles of the digital filter?
(The configuration of zeros and poles for the digital filter should look similar to the configuration of zeros and poles for the analog filter with the portion of the unit circle near $z$=1 corresponding to the $\omega$ axis in the $s$ plane.)

$$z^2 - 1.9991z + 1 = 0 \Rightarrow \text{Zeros at } z = 1\angle \pm 0.03$$
$$z^2 - 1.6667z + 0.6667 = 0 \Rightarrow \text{Poles at } z = 0.6667\angle 0 \text{ , } 1\angle 0$$

(e) What are the magnitude and phase of the digital filter frequency response at $\Omega = 0.08\pi$ and $\Omega = \pi$? What is the relationship between these digital filter frequency responses and those of the analog filter in part (b) above?

$$\mathrm{H}_d\left(e^{j0.08\pi}\right) = \mathrm{H}(1\angle 0.25133) = 0.953\frac{-1 - j1.9991 + 1}{-1 - j1.6667 + 0.6667} = 0.5265\angle 1.0074$$

$$\mathrm{H}_d\left(e^{j\pi}\right) = \mathrm{H}(-1) = 0.953\frac{1 + 1.9991 + 1}{1 + 1.6667 + 0.6667} = 1\angle 0$$