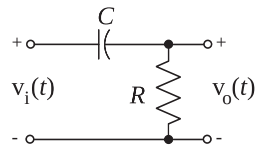
Solution of ECE 315 Test 11 F06

1. Below is a practical passive continuous-time filter. Let $C = 16\mu\text{F}$ and $R = 1000\Omega$.



(a) Find its transfer function $\text{H}(f)$ in terms of *R*, *C* and *f* as variables.

$$\text{H}(f) = \frac{R}{R + 1/j2\pi fC} = \frac{j2\pi fRC}{j2\pi fRC + 1}$$

(b) At what numerical frequency *f* is its transfer function magnitude a minimum and what are the numerical transfer function magnitude and phase at that frequency?

$$f_{\min} = \underline{0} \qquad |\text{H}(f_{\min})| = \underline{0} \qquad \measuredangle \text{H}(f_{\min}) = \underline{\text{undefined}}$$

(c) At what numerical frequency *f* is its transfer function magnitude a maximum and what are the numerical transfer function magnitude and phase at that frequency?

$$f_{\max} = \underline{\infty} \qquad |\text{H}(f_{\max})| = \underline{1} \qquad \measuredangle \text{H}(f_{\max}) = \underline{0}$$

(d) What are the magnitude and phase of the transfer function at a frequency of 10 Hz?

$|\text{H}(10)| = 0.709$ $\qquad\qquad$ $\measuredangle \text{H}(10) = 0.7828$ radians or $44.84°$

$$\text{H}(10) = \frac{j20\pi RC}{j20\pi RC + 1} \text{ and } RC = 0.016$$

$$\text{H}(10) = \frac{j20\pi(0.016)}{j20\pi(0.016) + 1} = 0.5026 + j0.5 = 0.709\measuredangle 0.7828$$

(e) If you keep $R = 1000\Omega$ and choose a new capacitor value *C* to make the magnitude of the transfer function at 100 Hz less than 30% of the maximum transfer function magnitude, what is the largest numerical value of *C* you could use?

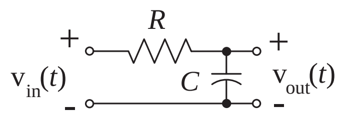
$$|\text{H}(100)|^2 = \left|\frac{j200000\pi C}{j200000\pi C + 1}\right|^2 = 0.3^2$$

$$\frac{4 \times 10^{10}\pi^2 C^2}{4 \times 10^{10}\pi^2 C^2 + 1} = 0.09$$

$$C^2 = \frac{0.09}{3.64 \times 10^{10}\pi^2} = 2.505 \times 10^{-13} \Rightarrow C = 0.5005\mu\text{F}$$

Solution of ECE 315 Test 11 F06

1. Below is a practical passive continuous-time filter. Let $C = 12\mu\text{F}$ and $R = 1000\Omega$.



(a) Find its transfer function $\text{H}(f)$ in terms of *R*, *C* and *f* as variables.

$$\text{H}(f) = \frac{1/j2\pi fC}{R + 1/j2\pi fC} = \frac{1}{j2\pi fRC + 1}$$

(b) At what numerical frequency *f* is its transfer function magnitude a minimum and what are the numerical transfer function magnitude and phase at that frequency?

$$f_{min} = \underline{\infty} \qquad |\text{H}(f_{min})| = \underline{0} \qquad \measuredangle\text{H}(f_{min}) = \underline{\text{undefined}}$$

(c) At what numerical frequency *f* is its transfer function magnitude a maximum and what are the numerical transfer function magnitude and phase at that frequency?

$$f_{max} = \underline{0} \qquad |\text{H}(f_{max})| = \underline{1} \qquad \measuredangle\text{H}(f_{max}) = \underline{0}$$

(d) What are the magnitude and phase of the transfer function at a frequency of 10 Hz?

$|\text{H}(10)| = 0.7985$ $\qquad$ $\measuredangle\text{H}(10) = $ -0.646 radians or 37.02°

$$\text{H}(10) = \frac{1}{j20\pi RC + 1} \text{ and } RC = 0.012$$

$$\text{H}(10) = \frac{1}{j20\pi(0.012) + 1} = 0.6376 - j0.4807 = 0.7985\measuredangle - 0.646$$

(e) If you keep $R = 1000\Omega$ and choose a new capacitor value *C* to make the magnitude of the transfer function at 100 Hz less than 40% of the maximum transfer function magnitude, what is the smallest numerical value of *C* you could use?

$$|\text{H}(100)|^2 = \left|\frac{1}{j200000\pi C + 1}\right|^2 = 0.4^2$$

$$\frac{1}{4\times10^{10}\pi^2 C^2 + 1} = 0.16$$

$$0.84 = 0.64\times10^{10}\pi^2 C^2 \Rightarrow C^2 = 1.33\times10^{-11} \Rightarrow C = 3.647\mu F$$