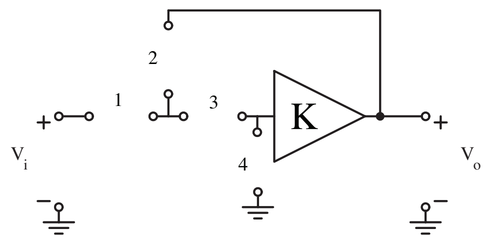
Solution of ECE 316 Test 1 Su09

1. Using only resistors and capacitors put single components into the circuit diagram below in the numbered positions that will make the frequency response of this filter $H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)}$ bandpass in nature with two poles. You need not put values on the components, just indicate whether they are capacitors or resistors. (There is more than one correct answer.)

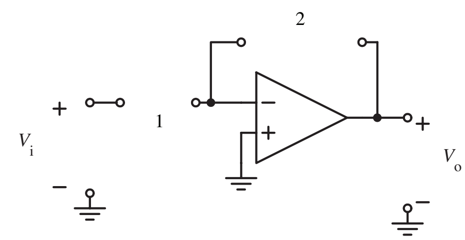


1 - Capacitor, 2-Resistor, 3-Resistor, 4-Capacitor

OR

1-Resistor, 2-Capacitor, 3-Capacitor, 4-Resistor

2. Using only resistors and capacitors put single components or a parallel or series combinations of two components into the circuit diagram below in the numbered positions that will make the frequency response of this filter $H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)}$ bandpass in nature with two poles. You need not put values on the components, just indicate whether they are capacitors or resistors or some combination of capacitors and resistors.



1 - Resistor and Capacitor in series, 2 - Resistor and Capacitor in Parallel

3. Referring to the system block diagram below

(a) Write the difference equations

i. relating $\mathrm{w}[n]$ to $\mathrm{x}[n]$ without reference to $\mathrm{v}[n]$,

$$\mathrm{w}[n] = \mathrm{x}[n] - \alpha\,\mathrm{w}[n-1]$$

and

ii. relating $\mathrm{v}[n]$ to $\mathrm{x}[n]$ without reference to $\mathrm{w}[n]$.

$$-\mathrm{v}[n+1] = \mathrm{w}[n] = \mathrm{x}[n] - \alpha \underbrace{\mathrm{w}[n-1]}_{=-\mathrm{v}[n]} \Rightarrow \mathrm{v}[n] = -\left(\alpha\,\mathrm{v}[n-1] + \mathrm{x}[n-1]\right)$$

(b) Then z transform the equations from part (a) and find the transfer function $\dfrac{\mathrm{Y}_1(z)}{\mathrm{X}(z)}$ without $\mathrm{W}(z)$ or $\mathrm{V}(z)$ appearing in it.

$$\mathrm{W}(z) = \mathrm{X}(z) - \alpha z^{-1}\,\mathrm{W}(z) \Rightarrow \mathrm{W}(z) = \frac{\mathrm{X}(z)}{1+\alpha z^{-1}}$$

$$\mathrm{V}(z) = -\left[\alpha z^{-1}\,\mathrm{V}(z) + z^{-1}\,\mathrm{X}(z)\right] \Rightarrow \mathrm{V}(z) = -\frac{z^{-1}\,\mathrm{X}(z)}{1+\alpha z^{-1}}$$

$$\mathrm{Y}_1(z) = \mathrm{W}(z) + \mathrm{V}(z) = \frac{\mathrm{X}(z)}{1+\alpha z^{-1}} - \frac{z^{-1}\,\mathrm{X}(z)}{1+\alpha z^{-1}} = \frac{z-1}{z+\alpha}\mathrm{X}(z)$$

$$\frac{\mathrm{Y}_1(z)}{\mathrm{X}(z)} = \frac{z-1}{z+\alpha}$$

For $\alpha = 0.8$ this stage of the overall filter should be highpass.

(c) Then find the transfer function $\frac{Y(z)}{Y_1(z)}$.

$$y[n+1] = y_1[n] - \beta y[n] \Rightarrow y[n] + \beta y[n-1] = y_1[n-1] \Rightarrow Y(z) = \frac{z^{-1} Y_1(z)}{1+\beta z^{-1}} = \frac{1}{z+\beta} Y_1(z)$$

$$\frac{Y(z)}{Y_1(z)} = \frac{1}{z+\beta}$$

For $\beta = 0.5$ this stage of the overall filter should not be obviously classifiable as highpass, lowpass, etc.

(d) The transfer function of this entire filter can be expressed in the form $H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}$. If $\alpha = 0.8$ and $\beta = 0.5$, find the numerical values of the constants.

b_0 = ____________ , b_1 = ____________ , b_2 = ____________

a_1 = ____________ , a_2 = ____________

You should have a highpass filter overall.

$$H(z) = \frac{Y_1(z)}{X(z)} \times \frac{Y(z)}{Y_1(z)} = \frac{z-1}{z+\alpha} \times \frac{1}{z+\beta} = \frac{z-1}{z^2 + (\alpha+\beta)z + \alpha\beta}$$

$$H(z) = \frac{z-1}{z^2 + 1.3z + 0.4} = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}$$