Solution of ECE 316 Test 2 S11

1. The signal $x(t) = 30\cos(2000\pi t)\sin(50\pi t)$ is sampled at a rate $f_s = 10^4$ samples/second with the first sample occurring at time $t = 0$. What is the numerical value of the third sample?

   0.2912

2. Find the numerical Nyquist rates of these signals. (If a signal is not bandlimited just write in "infinity" or "∞".)

   (a) $x(t) = 3\sin(3\times10^6\pi t) + 8\cos(2\times10^5\pi t)$

   $$X(f) = (j3/2)\left[\delta(f + 1.5\times10^6) - \delta(f - 1.5\times10^6)\right] + 4\left[\delta(f + 10^5) - \delta(f - 10^5)\right]$$

   Highest frequency is 1.5 MHz. Therefore the Nyquist rate is 3×10^6 samples/second.

   (b) $x(t) = 24\sin(3\times10^6\pi t)\cos(2\times10^5\pi t)$

   $$X(f) = j6\left[\delta(f + 1.5\times10^6) - \delta(f - 1.5\times10^6)\right] * \left[\delta(f - 10^5) + \delta(f + 10^5)\right]$$
   $$X(f) = j6\left[\delta(f + 1.4\times10^6) + \delta(f + 1.6\times10^6) - \delta(f - 1.6\times10^6) - \delta(f - 1.4\times10^6)\right]$$

   Highest frequency is 1.6 MHz. Therefore the Nyquist rate is 3.2×10^6 samples/second.

   (c) $x(t) = 4\,\mathrm{sinc}(t) * \delta_3(t)$

   $X(f) = (4/3)\mathrm{rect}(f)\delta_{1/3}(f) \Rightarrow$ Highest Frequency is at $f = 1/3 \Rightarrow$ Nyquist rate is $2/3$

   (d) $x(t) = \left[4\,\mathrm{sinc}(20t) * \delta_3(t)\right]\mathrm{tri}(t/10)$

   Time limited, therefore not bandlimited. Nyquist rate is infinite.

3. The continuous-time signal $x(t) = 10\cos(500\pi t)$ is sampled at a rate f_s to form a discrete-time signal $x[n]$. Find two different numerical sampling rates f_s that each yield a signal $x[n]$ with the maximum possible signal power.

If we sample at exactly the Nyquist rate, we sample at every positive and every negative peak. That makes the signal power as large as possible because every sample has a magnitude as large as possible. If we sample at a rate that is the same as the cosine's frequency we get a positive peak value every time and that also yields the maximum possible signal power. If we sample at any rate $250/n$ where n is any positive integer we also get only the positive peak values and the maximum signal power.

4. A system has a transfer function $H(s) = 3\dfrac{s^2+7s}{s^2+8s+4}$.

(a) In a magnitude Bode diagram of its frequency response what are the numerical values of all the corner frequencies (in radians/second)?

$$H(j\omega) = 3\frac{j\omega(j\omega+7)}{(j\omega+7.4641)(j\omega+0.5359)} \Rightarrow \text{Corners at } 0.5359, 7 \text{ and } 7.4641 \text{ radians/second}$$

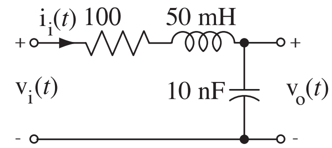
(b) What is the numerical slope (in dB/decade) of the magnitude Bode diagram at very low frequencies (approaching zero)?

$$\lim_{\omega\to 0} H(j\omega) = 3\frac{j7\omega}{7.4641\times 0.5359} = j5.25\omega \Rightarrow \text{Single differentiator} \Rightarrow \text{Slope} = +20\text{dB/decade}$$

(c) What is the numerical slope (in dB/decade) of the magnitude Bode diagram at very high frequencies (approaching infinity)?

$$\lim_{\omega\to 0} H(j\omega) = 3\frac{(j\omega)^2}{(j\omega)^2} = 3 \Rightarrow \text{Constant} \Rightarrow \text{Slope} = 0\text{dB/decade}$$

5. Referring to this circuit, answer the following questions.



(a) If the transfer function is defined as $H(s) = \frac{V_o(s)}{V_i(s)}$, what numerical value does the frequency response magnitude approach at very low frequencies (approaching zero) and at very high frequencies (approaching infinity)?

$\lim_{\omega \to 0} |H(j\omega)| = 1 \qquad \lim_{\omega \to \infty} |H(j\omega)| = 0$

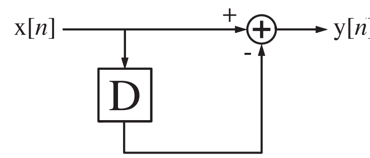
(b) If the transfer function is defined as $H(s) = \frac{V_i(s)}{I_i(s)}$, what is the minimum numerical value of the frequency response magnitude over all frequencies?

$$H(s) = R + sL + 1/sC \Rightarrow H(j\omega) = R + j\omega L + 1/j\omega C$$

$H(j\omega) = 100 + j0.05\omega - j10^8/\omega \Rightarrow$ Minimum occurs when the imaginary parts cancel

$$\min H(j\omega) = 100\Omega$$

6. Referring to this discrete-time filter,



(a) If $x[n] = u[n]$, fill in the blanks with numbers.

$y[0] = 1$ $y[1] = 0$ $y[33] = 0$

(b) If $x[n] = \delta[n]$, fill in the blanks with numbers.

$y[0] = 1$ $y[1] = -1$ $y[33] = 0$

(c) If $x[n] = \text{ramp}[n] = n\,u[n]$, fill in the blanks with numbers.

$y[0] = 0$ $y[1] = 1$ $y[33] = 1$

(d) If $x[n] = 10\cos(2\pi n / N_0)$ what value of N_0 makes $y[n]$ zero for all time?

If $N_0 = 2$ then $x[n] = 10\cos(\pi n) \Rightarrow y[n] = 0$

$y[n] = x[n] - x[n-1] \Rightarrow H(z) = 1 - z^{-1} \Rightarrow H(e^{j\Omega}) = 1 - e^{-j\Omega}$

To make $H(e^{j\Omega}) = 0$, set $1 - e^{-j\Omega} = 0 \Rightarrow \Omega = \pi = 2\pi / N_0 \Rightarrow N_0 = 2$

7. What is the main advantage of DSBTC modulation over DSBSC modulation in commercial AM radio broadcasting?

It allows the use of an asynchronous demodulator (envelope detector) in the receiver instead of a synchronous demodulator.

8. What is the main advantage of SSBSC modulation over DSBSC modulation?

It uses half the bandwidth to transmit the same information.

9. What is the numerical signal power of $x(t) = 5\cos(2\times10^5\pi t)\cos(2000\pi t)$?

$$x(t) = (5/2)\left[\cos(2\times10^5\pi t - 2000\pi t) + \cos(2\times10^5\pi t + 2000\pi t)\right]$$

$$P_x = \frac{(5/2)^2}{2} + \frac{(5/2)^2}{2} = 25/4$$

OR

$$X(f) = (5/4)\left[\delta(f - 10^5) + \delta(f + 10^5)\right] * \left[\delta(f - 1000) + \delta(f + 1000)\right]$$
$$X(f) = (5/4)\left[\delta(f - 101000) + \delta(f - 99000) + \delta(f + 99000) + \delta(f + 101000)\right]$$
$$P_x = (5/4)^2 + (5/4)^2 + (5/4)^2 + (5/4)^2 = 4\times25/16 = 25/4$$

Solution of ECE 316 Test 2 S11

1. The signal $x(t) = 30\cos(2100\pi t)\sin(50\pi t)$ is sampled at a rate $f_s = 10^4$ samples/second with the first sample occurring at time $t = 0$. What is the numerical value of the third sample?

   0.2343

2. Find the numerical Nyquist rates of these signals. (If a signal is not bandlimited just write in "infinity" or "∞".)

   (a) $x(t) = 3\sin(5\times10^6\pi t) + 8\cos(2\times10^5\pi t)$

   $$X(f) = (j3/2)\left[\delta(f+2.5\times10^6) - \delta(f-2.5\times10^6)\right] + 4\left[\delta(f+10^5) - \delta(f-10^5)\right]$$

   Highest frequency is 2.5 MHz. Therefore the Nyquist rate is 5×10^6 samples/second.

   (b) $x(t) = 24\sin(5\times10^6\pi t)\cos(2\times10^5\pi t)$

   $$X(f) = j6\left[\delta(f+2.5\times10^6) - \delta(f-2.5\times10^6)\right] * \left[\delta(f-10^5) + \delta(f+10^5)\right]$$
   $$X(f) = j6\left[\delta(f+2.4\times10^6) + \delta(f+2.6\times10^6) - \delta(f-2.6\times10^6) - \delta(f-2.4\times10^6)\right]$$

   Highest frequency is 2.6 MHz. Therefore the Nyquist rate is 5.2×10^6 samples/second.

   (c) $x(t) = 4\,\mathrm{sinc}(t) * \delta_5(t)$

   $X(f) = (4/5)\mathrm{rect}(f)\delta_{1/5}(f) \Rightarrow$ Highest Frequency is at $f = 2/5 \Rightarrow$ Nyquist rate is $4/5$

   (d) $x(t) = \left[4\,\mathrm{sinc}(20t) * \delta_5(t)\right]\mathrm{tri}(t/10)$

   Time limited, therefore not bandlimited. Nyquist rate is infinite.

3. The continuous-time signal $x(t) = 10\cos(700\pi t)$ is sampled at a rate f_s to form a discrete-time signal $x[n]$. Find two different numerical sampling rates f_s that each yield a signal $x[n]$ with the maximum possible signal power.

If we sample at exactly the Nyquist rate, we sample at every positive and every negative peak. That makes the signal power as large as possible because every sample has a magnitude as large as possible. If we sample at a rate that is the same as the cosine's frequency we get a positive peak value every time and that also yields the maximum possible signal power. If we sample at any rate $350 / n$ where n is any positive integer we also get only the positive peak values and the maximum signal power.

4. A system has a transfer function $H(s) = 3\dfrac{s+4}{s^2+9s+6}$.

(a) In a magnitude Bode diagram of its frequency response what are the numerical values of all the corner frequencies (in radians/second)?

$$H(j\omega) = 3\frac{j\omega+4}{(j\omega+8.2749)(j\omega+0.7251)} \Rightarrow \text{Corners at } 0.7251,\, 4 \text{ and } 8.7249 \text{ radians/second}$$

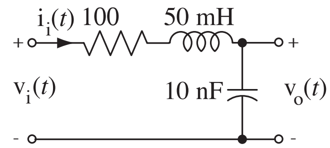
(b) What is the numerical slope (in dB/decade) of the magnitude Bode diagram at very low frequencies (approaching zero)?

$$\lim_{\omega\to 0} H(j\omega) = 3\frac{4}{6} = 2 \Rightarrow \text{Constant} \Rightarrow \text{Slope} = 0\text{dB/decade}$$

(c) What is the numerical slope (in dB/decade) of the magnitude Bode diagram at very high frequencies (approaching infinity)?

$$\lim_{\omega\to 0} H(j\omega) = 3\frac{j\omega}{(j\omega)^2} = 3/j\omega \Rightarrow \text{Slope} = -20\text{dB/decade}$$

5. Referring to this circuit, answer the following questions.



(a) If the transfer function is defined as $H(s) = \frac{V_o(s)}{V_i(s)}$, what numerical value does the frequency response magnitude approach at very low frequencies (approaching zero) and at very high frequencies (approaching infinity)?

$\lim_{\omega \to 0} |H(j\omega)| = 1 \qquad \lim_{\omega \to \infty} |H(j\omega)| = 0$

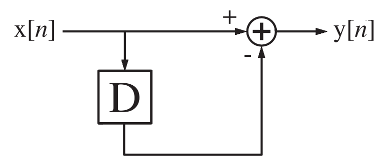
(b) If the transfer function is defined as $H(s) = \frac{V_i(s)}{I_i(s)}$, what is the minimum numerical value of the frequency response magnitude over all frequencies?

$H(s) = R + sL + 1/sC \Rightarrow H(j\omega) = R + j\omega L + 1/j\omega C$

$H(j\omega) = 100 + j0.05\omega - j10^8/\omega \Rightarrow$ Minimum occurs when the imaginary parts cancel

$\min H(j\omega) = 100\Omega$

6. Referring to this discrete-time filter,



(a) If $x[n] = u[n]$, fill in the blanks with numbers.

$y[0] = 1$ $y[1] = 0$ $y[33] = 0$

(b) If $x[n] = \delta[n]$, fill in the blanks with numbers.

$y[0] = 1$ $y[1] = -1$ $y[33] = 0$

(c) If $x[n] = \mathrm{ramp}[n] = n\,u[n]$, fill in the blanks with numbers.

$y[0] = 0$ $y[1] = 1$ $y[33] = 1$

(d) If $x[n] = 10\cos(2\pi n / N_0)$ what value of N_0 makes $y[n]$ zero for all time?

If $N_0 = 2$ then $x[n] = 10\cos(\pi n) \Rightarrow y[n] = 0$

$y[n] = x[n] - x[n-1] \Rightarrow H(z) = 1 - z^{-1} \Rightarrow H(e^{j\Omega}) = 1 - e^{-j\Omega}$

To make $H(e^{j\Omega}) = 0$, set $1 - e^{-j\Omega} = 0 \Rightarrow \Omega = \pi = 2\pi / N_0 \Rightarrow N_0 = 2$

7. What is the main advantage of DSBTC modulation over DSBSC modulation in commercial AM radio broadcasting?

It allows the use of an asynchronous demodulator (envelope detector) in the receiver instead of a synchronous demodulator.

8. What is the main advantage of SSBSC modulation over DSBSC modulation?

It uses half the bandwidth to transmit the same information.

9. What is the numerical signal power of $\mathrm{x}(t) = 12\cos\left(2\times 10^5 \pi t\right)\cos(2000\pi t)$?

$$\mathrm{x}(t) = 6\left[\cos\left(2\times 10^5 \pi t - 2000\pi t\right) + \cos\left(2\times 10^5 \pi t + 2000\pi t\right)\right]$$

$$P_{\mathrm{x}} = \frac{(6)^2}{2} + \frac{(6)^2}{2} = 36$$

OR

$$\mathrm{X}(f) = 3\left[\delta\left(f - 10^5\right) + \delta\left(f + 10^5\right)\right] * \left[\delta(f - 1000) + \delta(f + 1000)\right]$$
$$\mathrm{X}(f) = 3\left[\delta(f - 101000) + \delta(f - 99000) + \delta(f + 99000) + \delta(f + 101000)\right]$$
$$P_{\mathrm{x}} = 3^2 + 3^2 + 3^2 + 3^2 = 4\times 9 = 36$$

Solution of ECE 316 Test 2 S11

1. The signal $x(t) = 30\cos(1900\pi t)\sin(50\pi t)$ is sampled at a rate $f_s = 10^4$ samples/second with the first sample occurring at time $t = 0$. What is the numerical value of the third sample?

   0.3469

2. Find the numerical Nyquist rates of these signals. (If a signal is not bandlimited just write in "infinity" or "∞".)

   (a) $x(t) = 3\sin\left(4\times10^6\pi t\right) + 8\cos\left(2\times10^5\pi t\right)$

   $$X(f) = (j3/2)\left[\delta\left(f + 2\times10^6\right) - \delta\left(f - 2\times10^6\right)\right] + 4\left[\delta\left(f + 10^5\right) - \delta\left(f - 10^5\right)\right]$$

   Highest frequency is 2 MHz. Therefore the Nyquist rate is 4×10^6 samples/second.

   (b) $x(t) = 24\sin\left(4\times10^6\pi t\right)\cos\left(2\times10^5\pi t\right)$

   $$X(f) = j6\left[\delta\left(f + 2\times10^6\right) - \delta\left(f - 2\times10^6\right)\right] * \left[\delta\left(f - 10^5\right) + \delta\left(f + 10^5\right)\right]$$
   $$X(f) = j6\left[\delta\left(f + 1.9\times10^6\right) + \delta\left(f + 2.1\times10^6\right) - \delta\left(f - 2.1\times10^6\right) - \delta\left(f - 1.9\times10^6\right)\right]$$

   Highest frequency is 2.1 MHz. Therefore the Nyquist rate is 4.2×10^6 samples/second.

   (c) $x(t) = 4\,\mathrm{sinc}(t) * \delta_7(t)$

   $X(f) = (4/7)\mathrm{rect}(f)\delta_{1/7}(f) \Rightarrow$ Highest Frequency is at $f = 3/7 \Rightarrow$ Nyquist rate is $6/7$

   (d) $x(t) = \left[4\,\mathrm{sinc}(20t) * \delta_7(t)\right]\mathrm{tri}(t/10)$

   Time limited, therefore not bandlimited. Nyquist rate is infinite.

3. The continuous-time signal $x(t) = 10\cos(300\pi t)$ is sampled at a rate f_s to form a discrete-time signal $x[n]$. Find two different numerical sampling rates f_s that each yield a signal $x[n]$ with the maximum possible signal power.

If we sample at exactly the Nyquist rate, we sample at every positive and every negative peak. That makes the signal power as large as possible because every sample has a magnitude as large as possible. If we sample at a rate that is the same as the cosine's frequency we get a positive peak value every time and that also yields the maximum possible signal power. If we sample at any rate $150/n$ where n is any positive integer we also get only the positive peak values and the maximum signal power.

4. A system has a transfer function $H(s) = 3\dfrac{s}{(s^2+5s+2)(s+7)}$.

(a) In a magnitude Bode diagram of its frequency response what are the numerical values of all the corner frequencies (in radians/second)?

$$H(j\omega) = 3\frac{j\omega}{(j\omega+4.5616)(j\omega+0.4384)(j\omega+7)} \Rightarrow \text{Corners at } 0.4384, 4.5616 \text{ and } 7 \text{ radians/second}$$

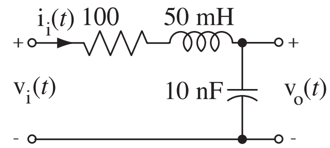
(b) What is the numerical slope (in dB/decade) of the magnitude Bode diagram at very low frequencies (approaching zero)?

$$\lim_{\omega\to 0} H(j\omega) = 3\frac{j\omega}{14} = j0.2143\omega \Rightarrow \text{Single differentiator} \Rightarrow \text{Slope} = +20\text{dB/decade}$$

(c) What is the numerical slope (in dB/decade) of the magnitude Bode diagram at very high frequencies (approaching infinity)?

$$\lim_{\omega\to 0} H(j\omega) = 3\frac{1}{(j\omega)^2} \Rightarrow \text{Slope} = -40\text{dB/decade}$$

5. Referring to this circuit, answer the following questions.



(a) If the transfer function is defined as $\mathrm{H}(s) = \frac{V_o(s)}{V_i(s)}$, what numerical value does the frequency response magnitude approach at very low frequencies (approaching zero) and at very high frequencies (approaching infinity)?

$\lim_{\omega \to 0} |\mathrm{H}(j\omega)| = 1 \qquad \lim_{\omega \to \infty} |\mathrm{H}(j\omega)| = 0$

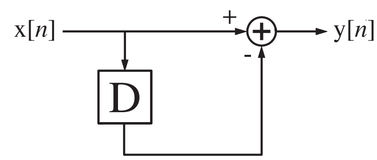
(b) If the transfer function is defined as $\mathrm{H}(s) = \frac{V_i(s)}{I_i(s)}$, what is the minimum numerical value of the frequency response magnitude over all frequencies?

$\mathrm{H}(s) = R + sL + 1/sC \Rightarrow \mathrm{H}(j\omega) = R + j\omega L + 1/j\omega C$

$\mathrm{H}(j\omega) = 100 + j0.05\omega - j10^8/\omega \Rightarrow$ Minimum occurs when the imaginary parts cancel

$\min \mathrm{H}(j\omega) = 100\Omega$

6. Referring to this discrete-time filter,



(a) If $\mathrm{x}[n]=\mathrm{u}[n]$, fill in the blanks with numbers.

$\mathrm{y}[0]=1$ $\mathrm{y}[1]=0$ $\mathrm{y}[33]=0$

(b) If $\mathrm{x}[n]=\delta[n]$, fill in the blanks with numbers.

$\mathrm{y}[0]=1$ $\mathrm{y}[1]=-1$ $\mathrm{y}[33]=0$

(c) If $\mathrm{x}[n]=\mathrm{ramp}[n]=n\,\mathrm{u}[n]$, fill in the blanks with numbers.

$\mathrm{y}[0]=0$ $\mathrm{y}[1]=1$ $\mathrm{y}[33]=1$

(d) If $\mathrm{x}[n]=10\cos(2\pi n/N_0)$ what value of N_0 makes $\mathrm{y}[n]$ zero for all time?

If $N_0=2$ then $\mathrm{x}[n]=10\cos(\pi n)\Rightarrow \mathrm{y}[n]=0$

$\mathrm{y}[n]=\mathrm{x}[n]-\mathrm{x}[n-1]\Rightarrow \mathrm{H}(z)=1-z^{-1}\Rightarrow \mathrm{H}(e^{j\Omega})=1-e^{-j\Omega}$

To make $\mathrm{H}(e^{j\Omega})=0$, set $1-e^{-j\Omega}=0\Rightarrow \Omega=\pi=2\pi/N_0\Rightarrow N_0=2$

7. What is the main advantage of DSBTC modulation over DSBSC modulation in commercial AM radio broadcasting?

It allows the use of an asynchronous demodulator (envelope detector) in the receiver instead of a synchronous demodulator.

8. What is the main advantage of SSBSC modulation over DSBSC modulation?

It uses half the bandwidth to transmit the same information.

9. What is the numerical signal power of $\mathrm{x}(t) = 3\cos\left(2\times10^5\pi t\right)\cos(2000\pi t)$?

$$\mathrm{x}(t) = (3/2)\left[\cos\left(2\times10^5\pi t - 2000\pi t\right) + \cos\left(2\times10^5\pi t + 2000\pi t\right)\right]$$

$$P_{\mathrm{x}} = \frac{(3/2)^2}{2} + \frac{(3/2)^2}{2} = 9/4$$

OR

$$\mathrm{X}(f) = (3/4)\left[\delta\left(f-10^5\right) + \delta\left(f+10^5\right)\right] * \left[\delta(f-1000) + \delta(f+1000)\right]$$
$$\mathrm{X}(f) = (3/4)\left[\delta(f-101000) + \delta(f-99000) + \delta(f+99000) + \delta(f+101000)\right]$$
$$P_{\mathrm{x}} = (3/4)^2 + (3/4)^2 + (3/4)^2 + (3/4)^2 = 4\times9/16 = 9/4$$