# Solution of ECE 316 Test 3 S11

1. Sketch a root locus for each of the diagrams below. The diagrams show the poles and zeros of the loop transfer function $\mathrm{T}(s)$ of a feedback system.

   For each system indicate whether it will be unstable at some finite positive value of the gain constant $K$

| | |
|---|---|
| A | Unstable at some finite positive $K$ |
| B | Stable for all finite positive $K$ |
| C | Stable for all finite positive $K$ |
| D | Unstable at some finite positive $K$ |
| E | Stable for all finite positive $K$ |
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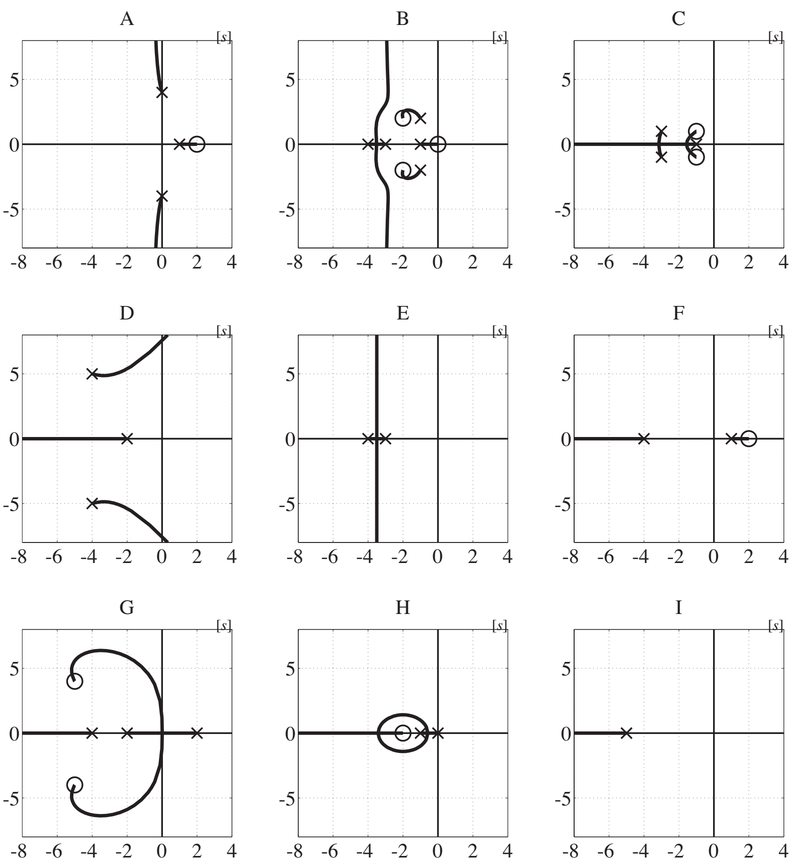

2. For each stable unity-gain feedback system whose forward-path transfer function $\mathrm{H}_1(s)$ is given below, indicate whether its error signal in response to a step and a ramp approaches zero, a finite, non-zero value or infinity as $t \to \infty$.

(a) $\mathrm{H}_1(s) = 1/s$

Step response error Zero
Ramp response error Finite, Non-zero

(b) $\mathrm{H}_1(s) = \dfrac{s}{s+2}$

Step response error Finite, Non-zero
Ramp response error Infinite

(c) $\mathrm{H}_1(s) = \dfrac{20}{s^2 + 3s + 40}$

Step response error Finite, Non-zero
Ramp response error Infinite

(d) $\mathrm{H}_1(s) = \dfrac{75}{s^3 + 9s^2 + 14s}$

Step response error Zero
Ramp response error Finite, Non-zero

(e) $\mathrm{H}_1(s) = \dfrac{200}{s^2(s+15)}$

Step response error Zero
Ramp response error Zero

3. We have studied 6 methods for approximating an analog filter by a digital filter.

Impulse Invariant, Step Invariant, Finite Difference, Matched-z, Direct Substitution and Bilinear

(a) Which method(s) can be done without using any time-domain functions in the process?

Finite Difference, Matched-z, Direct Substitution and Bilinear

(b) Which method(s) squeeze(s) the entire continuous-time frequency from zero to infinity into the discrete-time radian-frequency range zero to $\pi$ .

Bilinear

(c) Which two methods are almost the same, differing only by a time-domain delay?

Matched-z, Direct Substitution

(d) Which method can design an unstable digital filter while trying to approximate a stable analog filter and, if constrained to only stable designs, restricts the flexibility of the design to only certain types of filters?

Finite Difference

4. Using the bilinear method with a sampling rate of 10 samples/second find the transfer function of a digital filter that approximates an analog filter whose transfer function is

$$\mathrm{H}(s) = \frac{20}{s^2 + 3s + 8}.$$

The transfer function of the digital filter can be expressed in the form

$$\mathrm{H}(z) = A\frac{z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}.$$

Find the numerical values of the constants.

$A =$ ____________ , $b_1 =$ ____________ , $b_2 =$ ____________

$a_1 =$ ____________ , $a_2 =$ ____________

$$\mathrm{H}(z) = \frac{20}{\left(\frac{2}{T_s}\frac{z-1}{z+1}\right)^2 + 3\left(\frac{2}{T_s}\frac{z-1}{z+1}\right) + 8} = \frac{20T_s^2(z+1)^2}{4(z-1)^2 + 6T_s(z-1)(z+1) + 8T_s^2(z+1)^2}$$

$$\mathrm{H}(z) = \frac{20T_s^2 z^2 + 40T_s^2 z + 20T_s^2}{4z^2 - 8z + 4 + 6T_s z^2 - 6T_s + 8T_s^2 z^2 + 16T_s^2 z + 8T_s^2} = 20T_s^2 \frac{z^2 + 2z + 1}{\left(4 + 6T_s + 8T_s^2\right)z^2 - \left(8 - 16T_s^2\right)z + 8T_s^2 - 6T_s + 4}$$

$$\mathrm{H}(z) = 0.2\frac{z^2 + 2z + 1}{4.68z^2 - 7.84z + 3.48} = 0.0427\frac{z^2 + 2z + 1}{z^2 - 1.6752z + 0.7436}$$

5. Among the four analog filter types, Butterworth, Chebyshev Type 1, Chebyshev Type 2 and Elliptic, which one(s) have the following characteristics?

(a) Monotonic frequency response in both the passband and stopband.

Butterworth

(b) Monotonic frequency response in the passband and ripple in the stopband.

Chebyshev Type 2

(c) Monotonic frequency response in the stopband and ripple in the passband.

Chebyshev Type 1

(d) Ripple in both the passband and the stopband.

Elliptic

(e) The fastest transition from the passband to the stopband for a given filter order.

Elliptic

(f) The slowest transition from the passband to the stopband for a given filter order.

Butterworth

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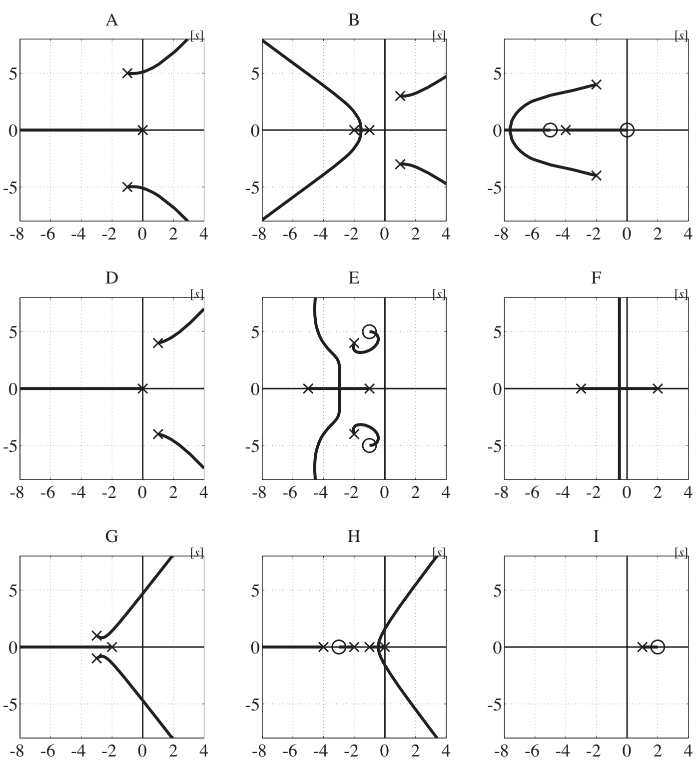

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(a) $H_1(s) = \dfrac{s}{s+2}$

Step response error Finite, Non-zero

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(b) $H_1(s) = \dfrac{20}{s^2 + 3s + 40}$

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(c) $H_1(s) = \dfrac{75}{s^3 + 9s^2 + 14s}$

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(a) Which method(s) squeeze(s) the entire continuous-time frequency from zero to infinity into the discrete-time radian-frequency range zero to $\pi$.

Bilinear

(b) Which two methods are almost the same, differing only by a time-domain delay?

Matched-z, Direct Substitution

(c) Which method can design an unstable digital filter while trying to approximate a stable analog filter and, if constrained to only stable designs, restricts the flexibility of the design to only certain types of filters?

Finite Difference

(d) Which method(s) can be done without using any time-domain functions in the process?

Finite Difference, Matched-z, Direct Substitution and Bilinear

4. Using the bilinear method with a sampling rate of 8 samples/second find the transfer function of a digital filter that approximates an analog filter whose transfer function is

$$\mathrm{H}(s) = \frac{20}{s^2 + 3s + 8} .$$

The transfer function of the digital filter can be expressed in the form

$$\mathrm{H}(z) = A\frac{z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} .$$

Find the numerical values of the constants.

$A =$ ____________ , $b_1 =$ ____________ , $b_2 =$ ____________

$a_1 =$ ____________ , $a_2 =$ ____________

$$\mathrm{H}(z) = \frac{20}{\left(\frac{2}{T_s}\frac{z-1}{z+1}\right)^2 + 3\left(\frac{2}{T_s}\frac{z-1}{z+1}\right) + 8} = \frac{20T_s^2(z+1)^2}{4(z-1)^2 + 6T_s(z-1)(z+1) + 8T_s^2(z+1)^2}$$

$$\mathrm{H}(z) = \frac{20T_s^2 z^2 + 40T_s^2 z + 20T_s^2}{4z^2 - 8z + 4 + 6T_s z^2 - 6T_s + 8T_s^2 z^2 + 16T_s^2 z + 8T_s^2} = 20T_s^2 \frac{z^2 + 2z + 1}{(4 + 6T_s + 8T_s^2)z^2 - (8 - 16T_s^2)z + 8T_s^2 - 6T_s + 4}$$

$$\mathrm{H}(z) = 0.0641\frac{z^2 + 2z + 1}{z^2 - 1.5897z + 0.6923}$$

5. Among the four analog filter types, Butterworth, Chebyshev Type 1, Chebyshev Type 2 and Elliptic, which one(s) have the following characteristics?

(a) Monotonic frequency response in the passband and ripple in the stopband.

Chebyshev Type 2

(b) Monotonic frequency response in the stopband and ripple in the passband.

Chebyshev Type 1

(c) Ripple in both the passband and the stopband.

Elliptic

(d) The fastest transition from the passband to the stopband for a given filter order.

Elliptic

(e) The slowest transition from the passband to the stopband for a given filter order.

Butterworth

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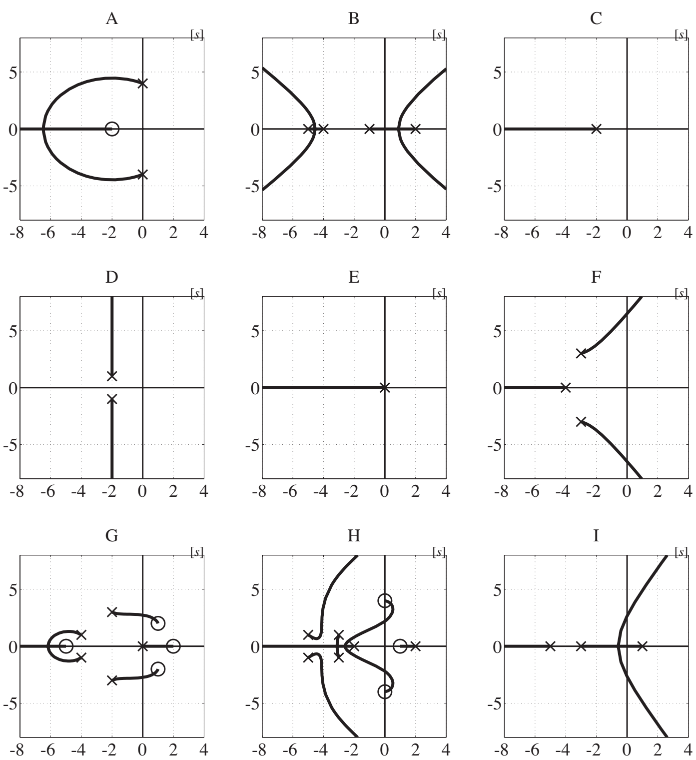

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Bilinear

(d) Which two methods are almost the same, differing only by a time-domain delay?

Matched-z, Direct Substitution

4. Using the bilinear method with a sampling rate of 12 samples/second find the transfer function of a digital filter that approximates an analog filter whose transfer function is

$$\mathrm{H}(s) = \frac{20}{s^2 + 3s + 8}\,.$$

The transfer function of the digital filter can be expressed in the form

$$\mathrm{H}(z) = A\frac{z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}\,.$$

Find the numerical values of the constants.

$A =$ ____________ , $b_1 =$ ____________ , $b_2 =$ ____________

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$$\mathrm{H}(z) = \frac{20}{\left(\frac{2}{T_s}\frac{z-1}{z+1}\right)^2 + 3\left(\frac{2}{T_s}\frac{z-1}{z+1}\right) + 8} = \frac{20T_s^2(z+1)^2}{4(z-1)^2 + 6T_s(z-1)(z+1) + 8T_s^2(z+1)^2}$$

$$\mathrm{H}(z) = \frac{20T_s^2 z^2 + 40T_s^2 z + 20T_s^2}{4z^2 - 8z + 4 + 6T_s z^2 - 6T_s + 8T_s^2 z^2 + 16T_s^2 z + 8T_s^2} = 20T_s^2\frac{z^2 + 2z + 1}{\left(4 + 6T_s + 8T_s^2\right)z^2 - \left(8 - 16T_s^2\right)z + 8T_s^2 - 6T_s + 4}$$

$$\mathrm{H}(z) = 0.0305\frac{z^2 + 2z + 1}{z^2 - 1.7317z + 0.7805}$$

5. Among the four analog filter types, Butterworth, Chebyshev Type 1, Chebyshev Type 2 and Elliptic, which one(s) have the following characteristics?

(a) The slowest transition from the passband to the stopband for a given filter order.

Butterworth

(b) Monotonic frequency response in both the passband and stopband.

Butterworth

(c) Monotonic frequency response in the passband and ripple in the stopband.

Chebyshev Type 2

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