

Solution of ECE 316 Test 1 Su09

1. The signal $x(t) = 4 \operatorname{tri}(t/4) * \delta_s(t)$ is sampled at a rate of $f_s = 2$ samples/second to form the discrete-time signal $x[n]$. The first sample is taken at time $t = 0$. Find the numerical values of

$$x(t) = 4 \sum_{k=-\infty}^{\infty} \operatorname{tri}((t - 9k)/4)$$

$$x[n] = 4 \sum_{k=-\infty}^{\infty} \operatorname{tri}((nT_s - 9k)/4) = 4 \sum_{k=-\infty}^{\infty} \operatorname{tri}((n/2 - 9k)/4) = 4 \sum_{k=-\infty}^{\infty} \operatorname{tri}((n - 18k)/8)$$

- (a) The second sample $x[1]$

$$x[1] = 4 \sum_{k=-\infty}^{\infty} \operatorname{tri}((1 - 18k)/8) = \dots + \underbrace{4 \operatorname{tri}(19/8)}_{k=-1 \Rightarrow 0} + \underbrace{4 \operatorname{tri}(1/8)}_{k=0 \Rightarrow 7/8} + \underbrace{4 \operatorname{tri}(-17/8)}_{k=+1 \Rightarrow 0} + \dots = 28/8 = 7/2 = 3.5$$

- (b) The 11th sample

$$x[10] = 4 \sum_{k=-\infty}^{\infty} \operatorname{tri}((10 - 18k)/8) = \dots + \underbrace{4 \operatorname{tri}(10/8)}_{k=0 \Rightarrow 0} + \underbrace{4 \operatorname{tri}(-8/8)}_{k=1 \Rightarrow 0} + \underbrace{4 \operatorname{tri}(-26/8)}_{k=2 \Rightarrow 0} + \dots = 0$$

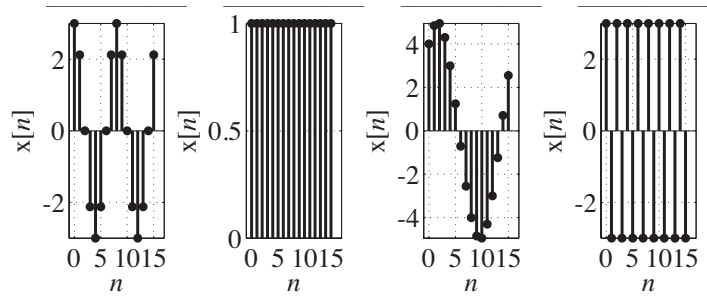
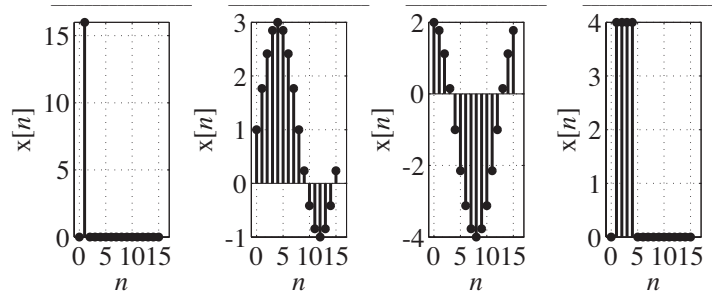
- (c) The 14th sample

$$x[13] = 4 \sum_{k=-\infty}^{\infty} \operatorname{tri}((13 - 18k)/8) = \dots + \underbrace{4 \operatorname{tri}(13/8)}_{k=0 \Rightarrow 0} + \underbrace{4 \operatorname{tri}(-5/8)}_{k=1 \Rightarrow 3/8} + \underbrace{4 \operatorname{tri}(-23/8)}_{k=2 \Rightarrow 0} + \dots = 12/8 = 3/2 = 1.5$$

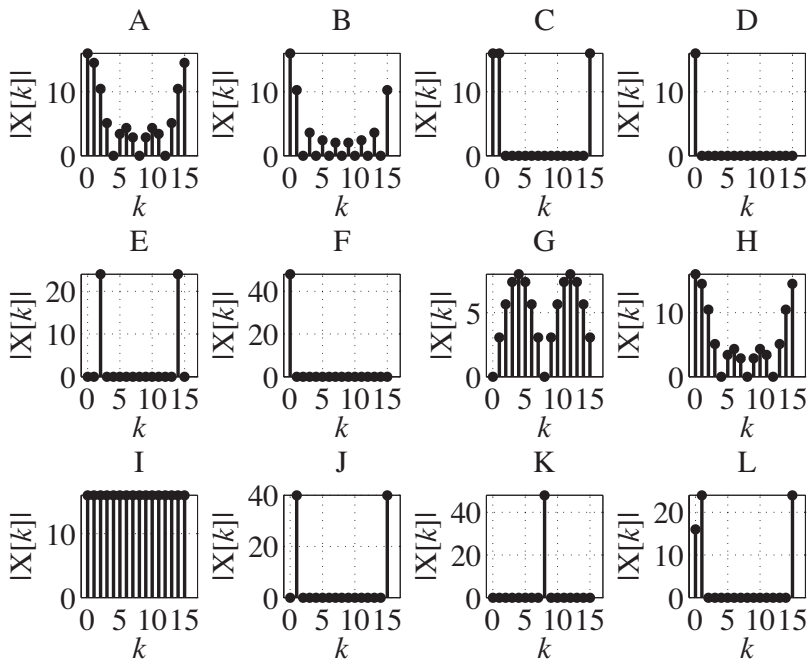
- (d) The 392nd sample

$$x[391] = 4 \sum_{k=-\infty}^{\infty} \operatorname{tri}((391 - 18k)/8) = \dots + \underbrace{4 \operatorname{tri}(13/8)}_{k=21 \Rightarrow 0} + \underbrace{4 \operatorname{tri}(-5/8)}_{k=22 \Rightarrow 3/8} + \underbrace{4 \operatorname{tri}(-23/8)}_{k=23 \Rightarrow 0} + \dots = 12/8 = 3/2 = 1.5$$

2. Enter in the blank space provided the letter designation of the magnitude DFT graph that corresponds to each discrete-time signal graph.



I C L A or H
E D J K



3. Find the numerical Nyquist rates for these signals. (If a signal is not bandlimited just enter "infinity" or " ∞ ".)

(a) $x(t) = 8 \operatorname{tri}((t - 4) / 12)$

$x(t) = 8 \operatorname{tri}((t - 4) / 12)$ is timelimited, therefore not bandlimited

(b) $x(t) = 3 \operatorname{sinc}((t - 4) / 5)$

$$X(f) = 15 \operatorname{rect}(5f) e^{-j8\pi f} \Rightarrow f_m = 1/10 \Rightarrow f_{N_{\text{Nyq}}} = 1/5$$

(c) $x(t) = -4 \sin(30\pi t) + 9 \cos(70\pi t)$

$$X(f) = -j2[\delta(f + 15) - \delta(f - 15)] + (9/2)[\delta(f - 35) + \delta(f + 35)] \Rightarrow f_m = 35 \Rightarrow f_{N_{\text{Nyq}}} = 70$$

(d) $x(t) = 22 \sin(30\pi t) \cos(70\pi t)$

$$X(f) = j(11/2)[\delta(f + 15) - \delta(f - 15)] * [\delta(f - 35) + \delta(f + 35)]$$

$$X(f) = j(11/2)[\delta(f - 20) + \delta(f + 50) - \delta(f - 50) - \delta(f + 20)] \Rightarrow f_m = 50 \Rightarrow f_{N_{\text{Nyq}}} = 100$$

(e) $x(t) = 13e^{-20t} \cos(70\pi t) u(t)$

$$X(f) = 13 \frac{j\omega + 20}{(j\omega + 20)^2 + (70\pi)^2} \Rightarrow \text{Not Bandlimited}$$

(f) $x(t) = 11 \operatorname{sinc}(10t) * \delta_{1/3}(t)$

$$X(f) = (11/10) \operatorname{rect}(f/10) (3) \delta_3(f) \Rightarrow f_m = 3 \Rightarrow f_{N_{\text{Nyq}}} = 6$$

Student Identification Number

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(No name please)

By entering my student identification number I affirm that I have neither given nor received help from anyone on this test.

EECS 316 Test 2 Su09

(Closed Book and Notes, Formula Sheet, 8.5 by 11 Sheet, Calculator, 1 hour)

If any question or problem is incomplete, ambiguous or contradictory, explain why. If you are correct you will receive full credit on that question or problem.

1. The signal $x(t) = 5 \text{tri}(t/5) * \delta_{11}(t)$ is sampled at a rate of $f_s = 2$ samples/second to form the discrete-time signal $x[n]$. The first sample is taken at time $t = 0$. Find the numerical values of

(a) (4 pts) The second sample $x[1]$ _____

(b) (5 pts) The 11th sample _____

(c) (5 pts) The 14th sample _____

(d) (6 pts) The 392nd sample _____

Solution of ECE 316 Test 1 Su09

1. The signal $x(t) = 5 \operatorname{tri}(t/5) * \delta_{11}(t)$ is sampled at a rate of $f_s = 2$ samples/second to form the discrete-time signal $x[n]$. The first sample is taken at time $t = 0$. Find the numerical values of

$$x(t) = 5 \sum_{k=-\infty}^{\infty} \operatorname{tri}((t - 11k)/5)$$

$$x[n] = 5 \sum_{k=-\infty}^{\infty} \operatorname{tri}((nT_s - 11k)/5) = 5 \sum_{k=-\infty}^{\infty} \operatorname{tri}((n/2 - 11k)/5) = 5 \sum_{k=-\infty}^{\infty} \operatorname{tri}((n - 22k)/10)$$

- (a) The second sample $x[1]$

$$x[1] = 5 \sum_{k=-\infty}^{\infty} \operatorname{tri}((1 - 22k)/10) = \dots + 5 \underbrace{\operatorname{tri}(23/10)}_{k=-1 \Rightarrow 0} + 5 \underbrace{\operatorname{tri}(1/10)}_{k=0 \Rightarrow 9/10} + 5 \underbrace{\operatorname{tri}(-21/10)}_{k=+1 \Rightarrow 0} + \dots = 45/10 = 4.5$$

- (b) The 11th sample

$$x[10] = 5 \sum_{k=-\infty}^{\infty} \operatorname{tri}((10 - 22k)/10) = \dots + 5 \underbrace{\operatorname{tri}(10/10)}_{k=0 \Rightarrow 0} + 5 \underbrace{\operatorname{tri}(-12/10)}_{k=1 \Rightarrow 0} + 5 \underbrace{\operatorname{tri}(-34/10)}_{k=2 \Rightarrow 0} + \dots = 0$$

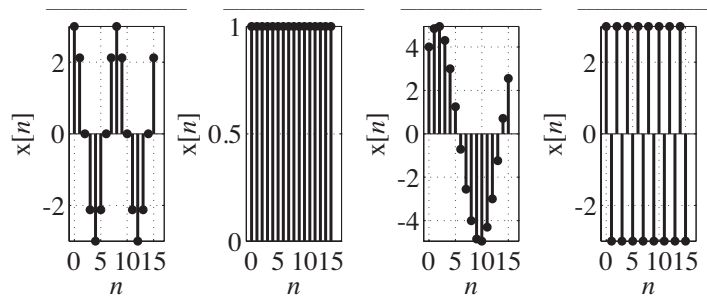
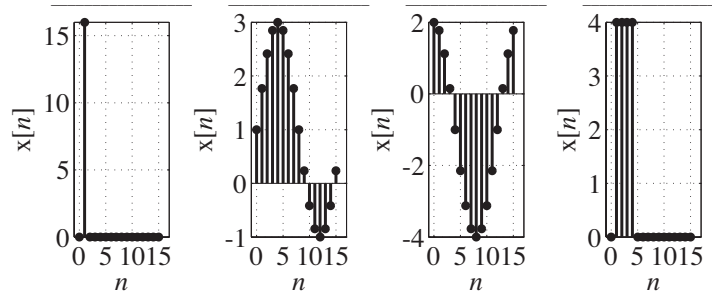
- (c) The 14th sample

$$x[13] = 5 \sum_{k=-\infty}^{\infty} \operatorname{tri}((13 - 22k)/10) = \dots + 5 \underbrace{\operatorname{tri}(13/10)}_{k=0 \Rightarrow 0} + 5 \underbrace{\operatorname{tri}(-9/10)}_{k=1 \Rightarrow 1/10} + 5 \underbrace{\operatorname{tri}(-31/10)}_{k=2 \Rightarrow 0} + \dots = 0.5$$

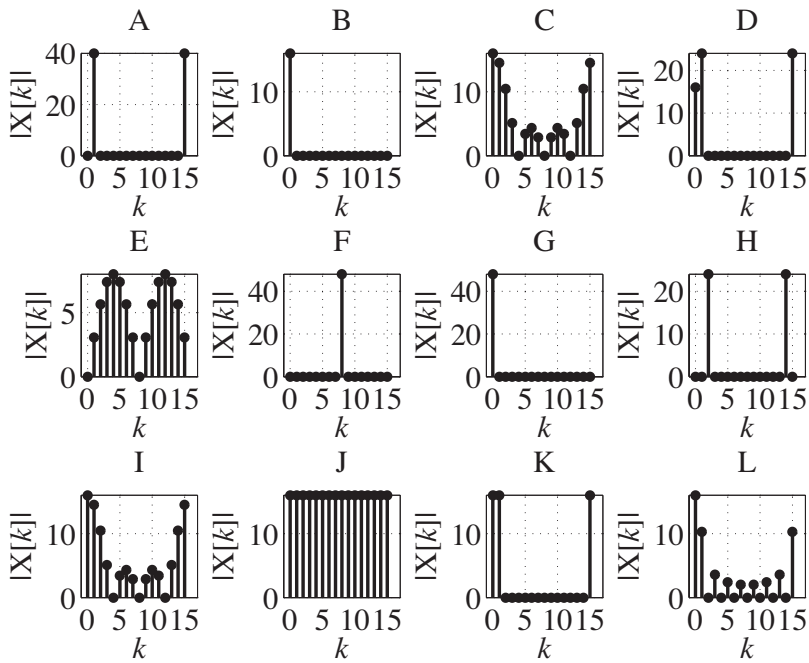
- (d) The 392nd sample

$$x[391] = 5 \sum_{k=-\infty}^{\infty} \operatorname{tri}((391 - 22k)/10) = \dots + 5 \underbrace{\operatorname{tri}(17/10)}_{k=17 \Rightarrow 0} + 5 \underbrace{\operatorname{tri}(-5/10)}_{k=18 \Rightarrow 1/2} + 5 \underbrace{\operatorname{tri}(-27/10)}_{k=19 \Rightarrow 0} + \dots = 5/2 = 2.5$$

2. Enter in the blank space provided the letter designation of the magnitude DFT graph that corresponds to each discrete-time signal graph.



J H K B D A I or C



3. Find the numerical Nyquist rates for these signals. (If a signal is not bandlimited just enter "infinity" or " ∞ ".)

(a) $x(t) = 8 \operatorname{tri}((t - 4) / 12)$

$x(t) = 8 \operatorname{tri}((t - 4) / 12)$ is timelimited, therefore not bandlimited

(b) $x(t) = 3 \operatorname{sinc}((t - 4) / 7)$

$$X(f) = 21 \operatorname{rect}(7f) e^{-j8\pi f} \Rightarrow f_m = 1/14 \Rightarrow f_{Nyq} = 1/7$$

(c) $x(t) = -4 \sin(30\pi t) + 9 \cos(90\pi t)$

$$X(f) = -j2[\delta(f + 15) - \delta(f - 15)] + (9/2)[\delta(f - 45) + \delta(f + 45)] \Rightarrow f_m = 45 \Rightarrow f_{Nyq} = 90$$

(d) $x(t) = 22 \sin(30\pi t) \cos(90\pi t)$

$$X(f) = j(11/2)[\delta(f + 15) - \delta(f - 15)] * [\delta(f - 45) + \delta(f + 45)]$$

$$X(f) = j(11/2)[\delta(f - 30) + \delta(f + 60) - \delta(f - 60) - \delta(f + 30)] \Rightarrow f_m = 60 \Rightarrow f_{Nyq} = 120$$

(e) $x(t) = 13e^{-20t} \cos(70\pi t) u(t)$

$$X(f) = 13 \frac{j\omega + 20}{(j\omega + 20)^2 + (70\pi)^2} \Rightarrow \text{Not Bandlimited}$$

(f) $x(t) = 11 \operatorname{sinc}(16t) * \delta_{1/3}(t)$

$$X(f) = (11/16) \operatorname{rect}(f/16)(3) \delta_3(f) \Rightarrow f_m = 6 \Rightarrow f_{Nyq} = 12$$

Solution of ECE 316 Test 1 Su09

1. The signal $x(t) = 8 \text{tri}(t/7) * \delta_{15}(t)$ is sampled at a rate of $f_s = 2$ samples/second to form the discrete-time signal $x[n]$. The first sample is taken at time $t = 0$. Find the numerical values of

$$x(t) = 8 \sum_{k=-\infty}^{\infty} \text{tri}((t - 15k)/7)$$

$$x[n] = 8 \sum_{k=-\infty}^{\infty} \text{tri}((nT_s - 15k)/7) = 8 \sum_{k=-\infty}^{\infty} \text{tri}((n/2 - 15k)/7) = 8 \sum_{k=-\infty}^{\infty} \text{tri}((n - 30k)/14)$$

- (a) The second sample $x[1]$

$$x[1] = 8 \sum_{k=-\infty}^{\infty} \text{tri}((1 - 30k)/14) = \dots + \underbrace{8 \text{tri}(31/14)}_{k=-1 \Rightarrow 0} + \underbrace{8 \text{tri}(1/14)}_{k=0 \Rightarrow 13/14} + \underbrace{8 \text{tri}(-29/14)}_{k=+1 \Rightarrow 0} + \dots = 104/14 = 7.43$$

- (b) The 11th sample

$$x[10] = 8 \sum_{k=-\infty}^{\infty} \text{tri}((10 - 30k)/14) = \dots + \underbrace{8 \text{tri}(40/14)}_{k=-1 \Rightarrow 0} + \underbrace{8 \text{tri}(-10/14)}_{k=0 \Rightarrow 4/14} + \underbrace{8 \text{tri}(-20/14)}_{k=1 \Rightarrow 0} + \dots = 16/7 = 2.29$$

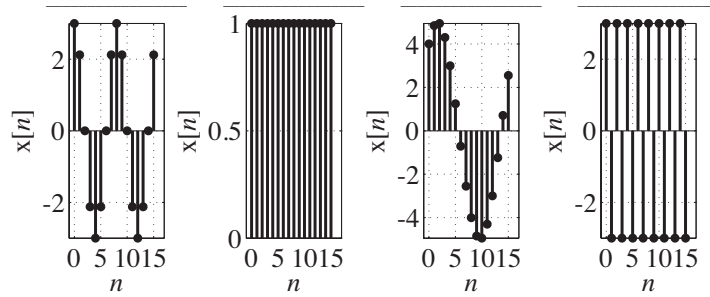
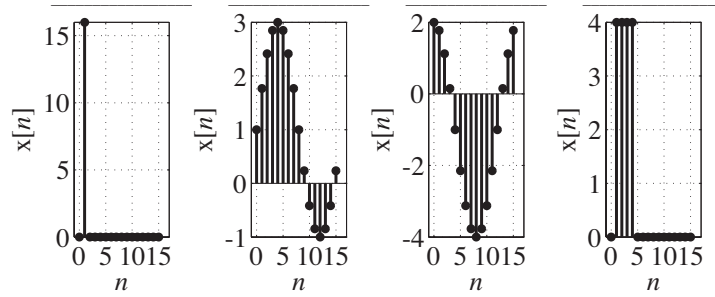
- (c) The 14th sample

$$x[13] = 8 \sum_{k=-\infty}^{\infty} \text{tri}((13 - 30k)/14) = \dots + \underbrace{8 \text{tri}(43/14)}_{k=-1 \Rightarrow 0} + \underbrace{8 \text{tri}(13/14)}_{k=0 \Rightarrow 1/14} + \underbrace{8 \text{tri}(-17/14)}_{k=1 \Rightarrow 0} + \dots = 8/14 = 0.571$$

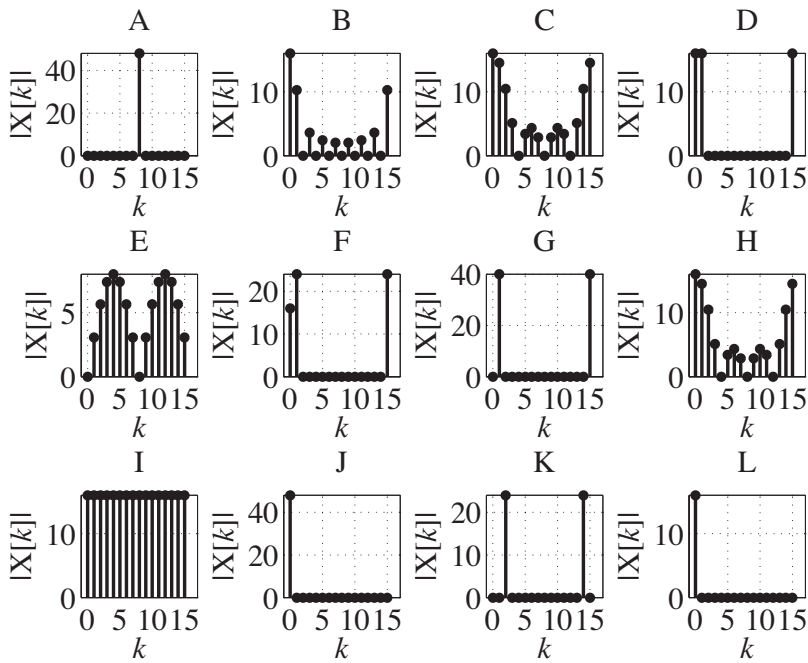
- (d) The 392nd sample

$$x[391] = 8 \sum_{k=-\infty}^{\infty} \text{tri}((391 - 30k)/14) = \dots + \underbrace{8 \text{tri}(31/14)}_{k=12 \Rightarrow 0} + \underbrace{8 \text{tri}(1/14)}_{k=13 \Rightarrow 13/14} + \underbrace{8 \text{tri}(-29/14)}_{k=14 \Rightarrow 0} + \dots = 104/14 = 7.43$$

2. Enter in the blank space provided the letter designation of the magnitude DFT graph that corresponds to each discrete-time signal graph.



I D F C or H
K L G A



3. Find the numerical Nyquist rates for these signals. (If a signal is not bandlimited just enter "infinity" or " ∞ ".)

(a) $x(t) = 8 \operatorname{tri}((t - 4) / 12)$

$x(t) = 8 \operatorname{tri}((t - 4) / 12)$ is timelimited, therefore not bandlimited

(b) $x(t) = 3 \operatorname{sinc}((t - 4) / 11)$

$$X(f) = 33 \operatorname{rect}(11f) e^{-j8\pi f} \Rightarrow f_m = 1/22 \Rightarrow f_{\text{Nyq}} = 1/11$$

(c) $x(t) = -4 \sin(30\pi t) + 9 \cos(110\pi t)$

$$X(f) = -j2[\delta(f + 15) - \delta(f - 15)] + (9/2)[\delta(f - 55) + \delta(f + 55)] \Rightarrow f_m = 55 \Rightarrow f_{\text{Nyq}} = 110$$

(d) $x(t) = 22 \sin(30\pi t) \cos(110\pi t)$

$$X(f) = j(11/2)[\delta(f + 15) - \delta(f - 15)] * [\delta(f - 55) + \delta(f + 55)]$$

$$X(f) = j(11/2)[\delta(f - 40) + \delta(f + 70) - \delta(f - 70) - \delta(f + 40)] \Rightarrow f_m = 70 \Rightarrow f_{\text{Nyq}} = 140$$

(e) $x(t) = 13e^{-20t} \cos(70\pi t) u(t)$

$$X(f) = 13 \frac{j\omega + 20}{(j\omega + 20)^2 + (70\pi)^2} \Rightarrow \text{Not Bandlimited}$$

(f) $x(t) = 11 \operatorname{sinc}(20t) * \delta_{1/3}(t)$

$$X(f) = (11/20) \operatorname{rect}(f/20)(3) \delta_3(f) \Rightarrow f_m = 9 \Rightarrow f_{\text{Nyq}} = 18$$