Solution of ECE 316 Test 3 Su09

Continuous-Time Systems

1. What is the difference between the way bilateral Laplace transforms are calculated and the way unilateral Laplace transforms are calculated?

   The lower limit of integration on the transform integral is changed from $-\infty$ to 0^-.

2. In the time-shifting property of the unilateral Laplace transform what limitations are put on the shift?

   The shift must be chosen such that a causal signal remains causal.

3. Given the unilateral Laplace transform pair $\delta(t) \xleftrightarrow{\mathscr{L}} 1$, what is the unilateral Laplace transform of the first derivative of a unit impulse occurring at time $t=0$?

$$\frac{d}{dt}(\delta(t)) \xleftrightarrow{\mathscr{L}} s(1) - \underbrace{\delta(0^-)}_{=0} = s$$

4. The term $\frac{d^2}{dt^2}\left[\mathrm{x}(t)\right]$ appears in a differential equation. What is its unilateral Laplace transform (symbolically in terms of $\mathrm{x}(t)$ and $\mathrm{X}(s)$)?

$$\frac{d^2}{dt^2}\left[\mathrm{x}(t)\right] \xleftrightarrow{\mathscr{L}} s^2\,\mathrm{X}(s) - s\,\mathrm{x}(0^-) - \left[\frac{d}{dt}\mathrm{x}(t)\right]_{t=0^-}$$

5. What is required of a system transfer function $\mathrm{H}(s)$ to represent a stable system?

   All its poles must lie in the open left half of the s plane.

6. A feedback system of the type represented in the text has a loop transfer function $\mathrm{T}(s) = \frac{K}{s+3}$ and a forward-path transfer function of K. What range of real values of K will make the feedback system unstable?

$$\mathrm{H}(s) = \frac{K}{1+\frac{K}{s+3}} = \frac{K(s+3)}{s+3+K} \Rightarrow \text{ Pole at } s = -3-K$$

$$-3-K \geq 0 \text{ for instability} \Rightarrow K \leq -3$$

7. Under what conditions can a feedback system produce an output signal without an input signal?

When it has a one or more poles on the ω axis or in the right half of the *s* plane.

8. A stable unity-gain feedback system has a forward-path transfer function with one pole at $s = 0$. All the other poles are at other locations in the open left half of the *s* plane. Describe the nature of the error signal as time approaches infinity for

(a) Step Excitation   Error signal approaches zero.

(b) Ramp Excitation   Error signal approaches a non-zero constant.

9. A feedback system has a loop transfer function with one finite zero in the left half of the *s* plane and an adjustable gain parameter *K*. What is the minimum number of finite poles of the loop transfer function in the left half of the *s* plane for which we know that the system must go unstable at some finite positive value of *K*? Explain.

Four. With four finite poles the asymptotes of the root locus are at $\pi / 3$, π and $5\pi / 3$ and two of the branches are guaranteed to enter the right half of the *s* plane at some finite value of *K*.

10. A system with one finite pole in the left half of the *s* plane and no finite zeros has a step response which approaches its final value on a 20 ms time constant. The pole is then shifted to a more negative real value on the σ axis. What happens to the time constant?

The time constant gets smaller. The approach to the final value is faster.

11. A system has two complex-conjugate finite poles in the left half of the s plane at angles of $\pm 30°$ with respect to the negative σ axis and no finite zeros.

(a) If we move those poles to an angle of $\pm 60°$ with respect to the negative σ axis without changing their distance from the origin, how does that affect the system's step response?

The step response will take longer to settle to its final value.

(b) If we keep the angles at $\pm 30°$ but move the poles closer to the origin, how does that affect the system's step response?

The step response ripple will occur more slowly because the natural radian frequency has been lowered.

12. If a system is excited by $\mathrm{x}(t) = \cos(10\pi t)\mathrm{u}(t)$ and the system's transfer function is $\mathrm{H}(s) = \dfrac{4}{s(s+6)}$,

(a) What is the numerical amplitude of the forced response of the system?

$$\left|\mathrm{H}(j10\pi)\right| = \left|\frac{4}{j10\pi(j10\pi + 6)}\right| = \frac{4}{10\pi\left|j10\pi + 6\right|} = 0.00398$$

(b) What is the numerical phase in radians (relative to the exciting cosine) of the forced response of the system?

$$\measuredangle \mathrm{H}(j10\pi) = \measuredangle \frac{4}{j10\pi(j10\pi + 6)} = \measuredangle 4 - \measuredangle j10\pi - \measuredangle(j10\pi + 6) = -2.923 \text{ radians}$$

Discrete-Time Systems

1. In the time-shifting property of the unilateral z transform what limitations are put on the shift?

None. Both positive and negative shifts are allowed although the ways they are treated are different.

2. Given the unilateral z transform pair $\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{z}{z-1}$, what is the unilateral z transform of the first backward difference of the unit sequence $3\mathrm{u}[n-2]$? (A first backward difference of a discrete-time signal at some time n is its value at that time minus its value at time $n-1$.)

$$3\left(\mathrm{u}[n-2] - \mathrm{u}[n-3]\right) \xleftrightarrow{\mathcal{Z}} 3\frac{z}{z-1}\left(z^{-2} - z^{-3}\right) = 3\frac{z^{-2}}{z-1}(z-1) = 3z^{-2}$$

3. What is required of a system transfer function $\mathrm{H}(z)$ to represent a stable system?

All its poles must lie in the open interior of the unit circle in the z plane.

4. A feedback system of the type represented in the text has a loop transfer function $\mathrm{T}(z) = \dfrac{K}{z - 0.4}$ and a forward-path transfer function of K.

(a) Is there a finite positive value of K that will make the system unstable? Explain.

Yes. Its root locus must go outside the unit circle to terminate on the single zero at infinity.

(b) If the forward-path transfer function is changed to Kz does that change the answer from what it was in part (a)? Explain.

The answer does change. Now there is a zero a the origin of the z plane and the root locus terminates there, never leaving the interior of the unit circle for any finite positive value of K.

5. A feedback system has a loop transfer function with one finite zero at $z = 0$, no poles on or outside the unit circle and an adjustable gain parameter K. What is the minimum number of finite poles of the loop transfer function inside the unit circle for which we know that the system must go unstable at some finite positive value of K? Explain.

Two. With two finite poles, there must be one zero at infinity and the root locus will go outside the unit circle at some finite positive value of K.

6. Using the mapping relationship $z = e^{sT_s}$ and a sampling rate $f_s = 10$, sketch, in the space provided below, the region of the z plane that corresponds to the region of the s plane bounded by $-5 < \sigma < 5$ and $-5\pi < \omega < 5\pi$.

The corresponding limits on z are $0.607 < |z| < 1.65$ and $-\pi / 2 < \measuredangle z < \pi / 2$.

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Continuous-Time Systems

1. What is the difference between the way bilateral Laplace transforms are calculated and the way unilateral Laplace transforms are calculated?

   The lower limit of integration on the transform integral is changed from $-\infty$ to 0^-.

2. In the time-scaling property of the unilateral Laplace transform what limitations are put on the scale factor?

   The scale factor must be chosen such that a causal signal remains causal.

3. Given the unilateral Laplace transform pair $\mathrm{u}(t) \xleftrightarrow{\mathscr{L}} 1/s$, what is the unilateral Laplace transform of the first derivative of a unit step occurring at time $t = 0$?

$$\frac{d}{dt}(\mathrm{u}(t)) \xleftrightarrow{\mathscr{L}} s(1/s) - \mathrm{u}(0^-) = 1$$

4. The term $\frac{d^2}{dt^2}\left[\mathrm{x}(t)\right]$ appears in a differential equation. What is its unilateral Laplace transform (symbolically in terms of $\mathrm{x}(t)$ and $\mathrm{X}(s)$)?

$$\frac{d^2}{dt^2}\left[\mathrm{x}(t)\right] \xleftrightarrow{\mathscr{L}} s^2\,\mathrm{X}(s) - s\,\mathrm{x}(0^-) - \left[\frac{d}{dt}\mathrm{x}(t)\right]_{t=0^-}$$

5. What is required of a system transfer function $\mathrm{H}(s)$ to represent a stable system?

   All its poles must lie in the open left half of the s plane.

6. A feedback system of the type represented in the text has a loop transfer function $\mathrm{T}(s) = \frac{K}{s+10}$ and a forward-path transfer function of K. What range of real values of K will make the feedback system unstable?

$$\mathrm{H}(s) = \frac{K}{1 + \frac{K}{s+10}} = \frac{K(s+10)}{s+K+10} \Rightarrow \text{Pole at } s = -10 - K$$

$$-10 - K \ge 0 \text{ for instability} \Rightarrow K \le -10$$

7. Under what conditions can a feedback system produce an output signal without an input signal?

When it has a one or more poles on the ω axis or in the right half of the s plane.

8. A stable unity-gain feedback system has a forward-path transfer function with no poles at $s = 0$. All the poles are at other locations in the open left half of the s plane. Describe the nature of the error signal as time approaches infinity for

(a) Step Excitation — Error signal approaches a non-zero constant.

(b) Ramp Excitation — Error signal approaches infinity.

9. A feedback system has a loop transfer function with one finite zero in the left half of the s plane and an adjustable gain parameter K. What is the minimum number of finite poles of the loop transfer function in the left half of the s plane for which we know that the system must go unstable at some finite positive value of K? Explain.

Four. With four finite poles the asymptotes of the root locus are at $\pi / 3$, π and $5\pi / 3$ and two of the branches are guaranteed to enter the right half of the s plane at some finite value of K.

10. A system with one finite pole in the left half of the s plane and no finite zeros has a step response which approaches its final value on a 20 ms time constant. The pole is then shifted to a more positive real value on the negative σ axis. What happens to the time constant?

The time constant gets larger. The approach to the final value is slower.

11. A system has two complex-conjugate finite poles in the left half of the s plane at angles of $\pm 30°$ with respect to the negative σ axis and no finite zeros.

(a) If we move those poles to an angle of $\pm 60°$ with respect to the negative σ axis without changing their distance from the origin, how does that affect the system's step response?

The step response will take longer to settle to its final value.

(b) If we keep the angles at $\pm 30°$ but move the poles farther from the origin, how does that affect the system's step response?

The step response overshoot and ripple will occur faster because the natural radian frequency has been increased.

12. If a system is excited by $\mathrm{x}(t) = \cos(10\pi t)\mathrm{u}(t)$ and the system's transfer function is $\mathrm{H}(s) = \dfrac{s}{s+6}$,

(a) What is the numerical amplitude of the forced response of the system?

$$\left|\mathrm{H}(j10\pi)\right| = \left|\frac{j10\pi}{j10\pi + 6}\right| = \frac{10\pi}{|j10\pi + 6|} = 0.9823$$

(b) What is the numerical phase in radians (relative to the exciting cosine) of the forced response of the system?

$$\measuredangle \mathrm{H}(j10\pi) = \measuredangle j10\pi - \measuredangle(j10\pi + 6) = 0.18871 \text{ radians}$$

Discrete-Time Systems

1. In the time-shifting property of the unilateral z transform what limitations are put on the shift?

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2. Given the unilateral z transform pair $\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{z}{z-1}$, what is the unilateral z transform of the first backward difference of the unit sequence $3\mathrm{u}[n-2]$? (A first backward difference of a discrete-time signal at some time n is its value at that time minus its value at time $n-1$.)

$$3(\mathrm{u}[n-2] - \mathrm{u}[n-3]) \xleftrightarrow{\mathcal{Z}} 3\frac{z}{z-1}\left(z^{-2} - z^{-3}\right) = 3\frac{z^{-2}}{z-1}(z-1) = 3z^{-2}$$

3. What is required of a system transfer function $\mathrm{H}(z)$ to represent a stable system?

All its poles must lie in the open interior of the unit circle in the z plane.

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(a) Is there a finite positive value of K that will make the system unstable? Explain.

Yes. Its root locus must go outside the unit circle to terminate on the single zero at infinity.

(b) If the forward-path transfer function is changed to Kz does that change the answer from what it was in part (a)? Explain.

The answer does change. Now there is a zero a the origin of the z plane and the root locus terminates there, never leaving the interior of the unit circle for any finite positive value of K.

5. A feedback system has a loop transfer function with one finite zero at $z = 0$, no poles on or outside the unit circle and an adjustable gain parameter K. What is the minimum number of finite poles of the loop transfer function inside the unit circle for which we know that the system must go unstable at some finite positive value of K? Explain.

Two. With two finite poles, there must be one zero at infinity and the root locus will go outside the unit circle at some finite positive value of K.

6. Using the mapping relationship $z = e^{sT_s}$ and a sampling rate $f_s = 10$, sketch, in the space provided below, the region of the z plane that corresponds to the region of the s plane bounded by $-8 < \sigma < 8$ and $-10\pi < \omega < 0$.

The corresponding limits on z are $0.4493 < |z| < 2.2255$ and $-\pi < \measuredangle z < 0$.