Solution of ECE 316 Test 1 S10

1. In the filter and Bode diagram of its frequency response magnitude below, the frequency response of the filter is defined as $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ and the operational amplifier is ideal.

(a) If the transfer function $H(s)$ is expressed in the form

$$H(s) = A\frac{1}{s + 1/\tau}$$

find A and τ expressed in terms of R_f , R_i and C .

The transfer function is

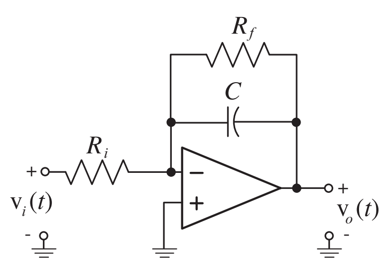
$$H(s) = -\frac{\frac{R_f / sC}{R_f + 1/sC}}{R_i} = -\frac{\frac{R_f}{sCR_f + 1}}{R_i} = -\frac{R_f}{R_i}\frac{1}{sCR_f + 1} = -\frac{1}{R_iC}\frac{1}{s + 1/R_fC}$$

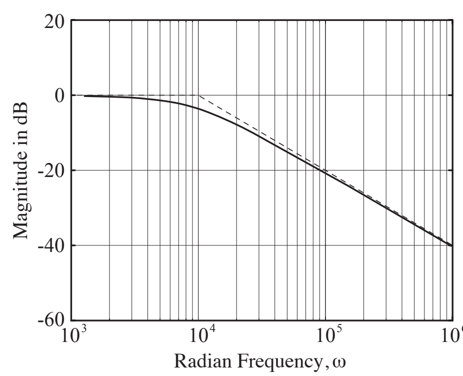
(b) If $R_f = 10\ \text{k}\Omega$ find the numerical values of R_i and C .

The corner frequency is 10,000 radians/second. $H(j\omega) = -\frac{1}{R_iC}\frac{1}{j\omega + 1/R_fC}$. Therefore

$$1/R_fC = 10{,}000 \Rightarrow C = 1/10{,}000R_f = 1/10^8 = 10\ \text{nF} .$$

The low-frequency gain is $-R_f / R_i$. Therefore, since from the Bode diagram it is also 0 dB, $-R_f / R_i = -1$ and $R_i = R_f = 10{,}000\ \Omega$.





2. A continuous-time system has a transfer function $\mathrm{H}(s) = \frac{s+1}{s(s+4)}$.

(a) At very low frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very low frequencies $\mathrm{H}(j\omega) \cong \frac{1}{j4\omega}$ and the system is approximately an integrator, the slope is -20 dB/decade and the phase is $-\pi/2$ radians.

(b) At very high frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very high frequencies $\mathrm{H}(j\omega) \cong \frac{j\omega}{(j\omega)^2} = \frac{1}{j\omega}$ and the system is again approximately an integrator, the slope is -20 dB/decade and the phase is $-\pi/2$ radians.

Change the transfer function to $\mathrm{H}(s) = \frac{s-1}{s(s+4)}$ and repeat parts (a) and (b).

(a) repeated

At very low frequencies $\mathrm{H}(j\omega) \cong \frac{-1}{j4\omega}$ and the system is approximately an integrator, the slope is -20 dB/decade and the phase is $\pi/2$ radians.

(b) repeated

At very high frequencies $\mathrm{H}(j\omega) \cong \frac{j\omega}{(j\omega)^2} = \frac{1}{j\omega}$ and the system is again approximately an integrator, the slope is -20 dB/decade and the phase is $-\pi/2$ radians.

3. A continuous-time system has a transfer function $\mathrm{H}(s)=\dfrac{1}{s^2+3s+20}$.

(a) At very low frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very low frequencies $\mathrm{H}(j\omega)\cong\dfrac{1}{20}$ and the system is approximately a frequency-independent gain, the slope is 0 dB/decade and the phase is 0 radians.

(b) At very high frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very high frequencies $\mathrm{H}(j\omega)\cong\dfrac{1}{(j\omega)^2}$ and the system is approximately a double integrator, the slope is -40 dB/decade and the phase is $-\pi$ radians.

4. A digital filter has a transfer function $\mathrm{H}(z)=\dfrac{z}{z-0.5}$. Its excitation is $\mathrm{x}[n]=\cos(2\pi n/N)$ and its response can be expressed in the form $\mathrm{y}[n]=A\cos(2\pi n/N-\theta)$.

(a) At what numerical radian frequency $-\pi\le\Omega<\pi$ is the amplitude of $\mathrm{y}[n]$ a maximum? ($\mathrm{H}(e^{j\Omega})=\mathrm{H}(z)\big|_{z\to e^{j\Omega}}$). (Be sure to carefully observe the limits and inequalities in $-\pi\le\Omega<\pi$.)

$$\mathrm{H}(e^{j\Omega})=\frac{e^{j\Omega}}{e^{j\Omega}-0.5}\Rightarrow\left|\mathrm{H}(e^{j\Omega})\right|=\frac{1}{\left|e^{j\Omega}-0.5\right|}$$

The maximum magnitude occurs at $\Omega=0$ and is $\left|\mathrm{H}(e^{j0})\right|=\dfrac{1}{|1-0.5|}=2$.

(b) At what numerical radian frequency $-\pi\le\Omega<\pi$ is the amplitude of $\mathrm{y}[n]$ a minimum?

The maximum magnitude occurs at $\Omega=-\pi$ and is $\left|\mathrm{H}(e^{-j\pi})\right|=\dfrac{1}{|-1-0.5|}=2/3$.

(c) If $N=4$, what is the numerical value of θ in radians?

$$\theta=\measuredangle\mathrm{H}(e^{j\Omega})\ ,\Omega=2\pi/N=\pi/2\Rightarrow\theta=\measuredangle\left(\frac{e^{j\pi/2}}{e^{j\pi/2}-0.5}\right)=\pi/2-\measuredangle\left(e^{j\pi/2}-0.5\right)$$

$$\theta=\pi/2-\measuredangle(j-0.5)=\pi/2-2.0344=-0.4636\text{ radians}$$

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1. In the filter and Bode diagram of its frequency response magnitude below, the frequency response of the filter is defined as $\mathrm{H}(j\omega)=\dfrac{V_o(j\omega)}{V_{\mathrm{i}}(j\omega)}$ and the operational amplifier is ideal.

(a) If the transfer function $\mathrm{H}(s)$ is expressed in the form

$$\mathrm{H}(s)=A\frac{1}{s+1/\tau}$$

find A and τ expressed in terms of R_f , R_i and C .

The transfer function is

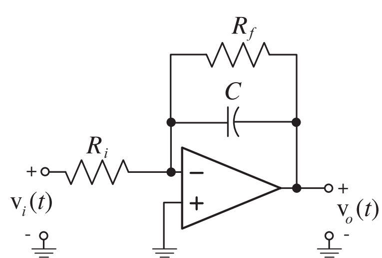
$$\mathrm{H}(s)=-\frac{\dfrac{R_f/sC}{R_f+1/sC}}{R_i}=-\frac{\dfrac{R_f}{sCR_f+1}}{R_i}=-\frac{R_f}{R_i}\frac{1}{sCR_f+1}=-\frac{1}{R_iC}\frac{1}{s+1/R_fC}$$

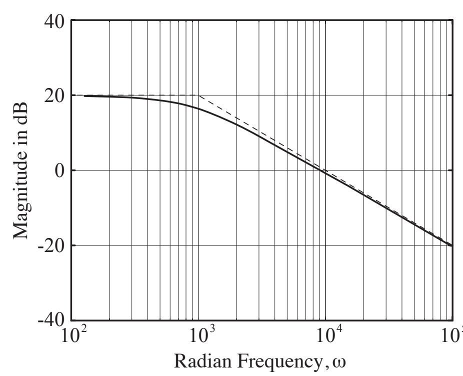
(b) If $R_f=10\ \mathrm{k}\Omega$ find the numerical values of R_i and C .

The corner frequency is 1,000 radians/second. $\mathrm{H}(j\omega)=-\dfrac{1}{R_iC}\dfrac{1}{j\omega+1/R_fC}$. Therefore

$$1/R_fC=1{,}000\Rightarrow C=1/1{,}000R_f=1/10^7=100\ \mathrm{nF}.$$

The low-frequency gain is $-R_f/R_i$. Therefore, since from the Bode diagram it is also 20 dB, $-R_f/R_i=-10$ and $R_i=R_f/10=1{,}000\ \Omega$.





2. A continuous-time system has a transfer function $\mathrm{H}(s)=\dfrac{s+1}{(s+10)(s+4)}$.

(a) At very low frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very low frequencies $\mathrm{H}(j\omega)\cong\dfrac{1}{40}$ and the system is approximately a frequency-independent gain, the slope is 0 dB/decade and the phase is 0 radians.

(b) At very high frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very high frequencies $\mathrm{H}(j\omega)\cong\dfrac{j\omega}{(j\omega)^2}=\dfrac{1}{j\omega}$ and the system is again approximately an integrator, the slope is -20 dB/decade and the phase is $-\pi/2$ radians.

Change the transfer function to $\mathrm{H}(s)=\dfrac{s-1}{(s+10)(s+4)}$ and repeat parts (a) and (b).

(a) repeated

At very low frequencies $\mathrm{H}(j\omega)\cong\dfrac{-1}{40}$ and the system is approximately a frequency-independent gain, the slope is 0 dB/decade and the phase is $\pm\pi$ radians.

(b) repeated

At very high frequencies $\mathrm{H}(j\omega)\cong\dfrac{j\omega}{(j\omega)^2}=\dfrac{1}{j\omega}$ and the system is again approximately an integrator, the slope is -20 dB/decade and the phase is $-\pi/2$ radians.

3. A continuous-time system has a transfer function $\mathrm{H}(s) = \frac{s}{s^2 + 3s + 20}$.

(a) At very low frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very low frequencies $\mathrm{H}(j\omega) \cong \frac{j\omega}{20}$ and the system is approximately a differentiator , the slope is +20 dB/decade and the phase is $\pi / 2$ radians.

(b) At very high frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very high frequencies $\mathrm{H}(j\omega) \cong \frac{1}{j\omega}$ and the system is approximately a double integrator, the slope is -20 dB/decade and the phase is $-\pi / 2$ radians.

4. A digital filter has a transfer function $\mathrm{H}(z) = \frac{z}{z - 0.3}$. Its excitation is $\mathrm{x}[n] = \cos(2\pi n / N)$ and its response can be expressed in the form $\mathrm{y}[n] = A\cos(2\pi n / N - \theta)$.

(a) At what numerical radian frequency $-\pi \le \Omega < \pi$ is the amplitude of $\mathrm{y}[n]$ a maximum? ($\mathrm{H}(e^{j\Omega}) = \mathrm{H}(z)\big|_{z \to e^{j\Omega}}$). (Be sure to carefully observe the limits and inequalities in $-\pi \le \Omega < \pi$.)

$$\mathrm{H}(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega} - 0.3} \Rightarrow \left|\mathrm{H}(e^{j\Omega})\right| = \frac{1}{\left|e^{j\Omega} - 0.3\right|}$$

The maximum magnitude occurs at $\Omega = 0$ and is $\left|\mathrm{H}(e^{j0})\right| = \frac{1}{|1 - 0.3|} = 0.1429$.

(b) At what numerical radian frequency $-\pi \le \Omega < \pi$ is the amplitude of $\mathrm{y}[n]$ a minimum?

The maximum magnitude occurs at $\Omega = -\pi$ and is $\left|\mathrm{H}(e^{-j\pi})\right| = \frac{1}{|-1 - 0.3|} = 0.769$.

(c) If $N = 4$, what is the numerical value of θ in radians?

$$\theta = \measuredangle \mathrm{H}(e^{j\Omega}) \text{ , } \Omega = 2\pi / N = \pi / 2 \Rightarrow \theta = \measuredangle\left(\frac{e^{j\pi/2}}{e^{j\pi/2} - 0.3}\right) = \pi / 2 - \measuredangle\left(e^{j\pi/2} - 0.3\right)$$

$$\theta = \pi / 2 - \measuredangle(j - 0.3) = \pi / 2 - 1.8623 = -0.2915 \text{ radians}$$

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(a) If the transfer function $\mathrm{H}(s)$ is expressed in the form

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The transfer function is

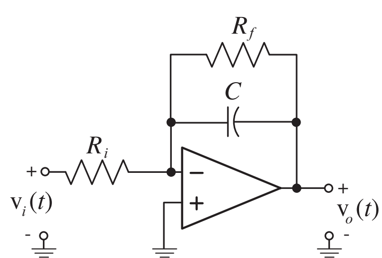
$$\mathrm{H}(s) = -\frac{\frac{R_f / sC}{R_f + 1/sC}}{R_i} = -\frac{\frac{R_f}{sCR_f + 1}}{R_i} = -\frac{R_f}{R_i}\frac{1}{sCR_f + 1} = -\frac{1}{R_i C}\frac{1}{s + 1/R_f C}$$

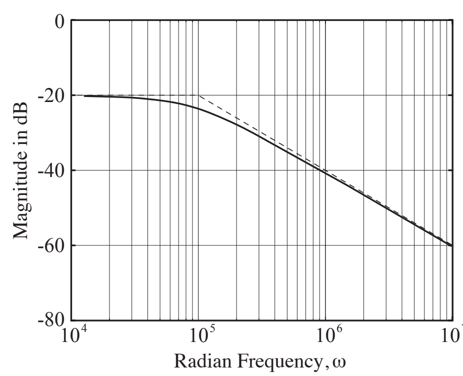
(b) If $R_f = 10\ \text{k}\Omega$ find the numerical values of R_i and C .

The corner frequency is 100,000 radians/second. $\mathrm{H}(j\omega) = -\frac{1}{R_i C}\frac{1}{j\omega + 1/R_f C}$. Therefore

$$1/R_f C = 100{,}000 \Rightarrow C = 1/100{,}000 R_f = 1/10^9 = 1\ \text{nF}\ .$$

The low-frequency gain is $-R_f / R_i$. Therefore, since from the Bode diagram it is also -20 dB, $-R_f / R_i = -0.1$ and $R_i = 10 R_f = 100{,}000\ \Omega$.





2. A continuous-time system has a transfer function $\mathrm{H}(s)=\frac{s+1}{s^2(s+4)}$.

(a) At very low frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very low frequencies $\mathrm{H}(j\omega)\cong\frac{1}{4(j\omega)^2}$ and the system is approximately a double integrator, the slope is -40 dB/decade and the phase is $\pm\pi$ radians.

(b) At very high frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very high frequencies $\mathrm{H}(j\omega)\cong\frac{j\omega}{(j\omega)^3}=\frac{1}{(j\omega)^2}$ and the system is again approximately a double integrator, the slope is -40 dB/decade and the phase is $\pm\pi$ radians.

Change the transfer function to $\mathrm{H}(s)=\frac{s-1}{s^2(s+4)}$ and repeat parts (a) and (b).

(a) repeated

At very low frequencies $\mathrm{H}(j\omega)\cong\frac{-1}{4(j\omega)^2}$ and the system is approximately a double-integrator, the slope is -40 dB/decade and the phase is 0 radians.

(b) repeated

At very high frequencies $\mathrm{H}(j\omega)\cong\frac{j\omega}{(j\omega)^3}=\frac{1}{(j\omega)^2}$ and the system is again approximately a double-integrator, the slope is -40 dB/decade and the phase is $\pm\pi$ radians.

3. A continuous-time system has a transfer function $\mathrm{H}(s) = \dfrac{s^2}{s^2 + 3s + 20}$.

(a) At very low frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very low frequencies $\mathrm{H}(j\omega) \cong \dfrac{(j\omega)^2}{20}$ and the system is approximately a doubledifferentiator, the slope is +40 dB/decade and the phase is $\pm\pi$ radians.

(b) At very high frequencies what slope (in dB/decade) does the Bode diagram of the magnitude frequency response approach and what phase angle (in radians) does the phase approach?

At very high frequencies $\mathrm{H}(j\omega) \cong 1$ and the system is approximately a frequency-independent gain, the slope is 0 dB/decade and the phase is 0 radians.

4. A digital filter has a transfer function $\mathrm{H}(z) = \dfrac{z}{z - 0.8}$. Its excitation is $\mathrm{x}[n] = \cos(2\pi n / N)$ and its response can be expressed in the form $\mathrm{y}[n] = A\cos(2\pi n / N - \theta)$.

(a) At what numerical radian frequency $-\pi \le \Omega < \pi$ is the amplitude of $\mathrm{y}[n]$ a maximum? ($\mathrm{H}(e^{j\Omega}) = \mathrm{H}(z)\big|_{z \to e^{j\Omega}}$). (Be sure to carefully observe the limits and inequalities in $-\pi \le \Omega < \pi$.)

$$\mathrm{H}(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega} - 0.8} \Rightarrow \left|\mathrm{H}(e^{j\Omega})\right| = \frac{1}{\left|e^{j\Omega} - 0.8\right|}$$

The maximum magnitude occurs at $\Omega = 0$ and is $\left|\mathrm{H}(e^{j0})\right| = \dfrac{1}{|1 - 0.8|} = 5$.

(b) At what numerical radian frequency $-\pi \le \Omega < \pi$ is the amplitude of $\mathrm{y}[n]$ a minimum?

The maximum magnitude occurs at $\Omega = -\pi$ and is $\left|\mathrm{H}(e^{-j\pi})\right| = \dfrac{1}{|-1 - 0.8|} = 0.5556$.

(c) If $N = 4$, what is the numerical value of θ in radians?

$$\theta = \measuredangle \mathrm{H}(e^{j\Omega}) \text{ , } \Omega = 2\pi / N = \pi / 2 \Rightarrow \theta = \measuredangle\left(\frac{e^{j\pi/2}}{e^{j\pi/2} - 0.8}\right) = \pi / 2 - \measuredangle\left(e^{j\pi/2} - 0.8\right)$$

$$\theta = \pi / 2 - \measuredangle(j - 0.5) = \pi / 2 - 2.2455 = -0.6747 \text{ radians}$$