Solution of ECE 316 Test 2 S10

1. Find the numerical Nyquist rates of these signals. (If a signal is not bandlimited, just write "Infinity" or "∞".)

(a) $\mathrm{x}(t) = \cos(24\pi t)\sin(80\pi t)$

$$\mathrm{X}(f) = (1/2)[\delta(f-12)+\delta(f+12)] * (j/2)[\delta(f+40)-\delta(f-40)]$$

$$\mathrm{X}(f) = (j/4)[\delta(f+28)-\delta(f-52)+\delta(f+52)-\delta(f-28)]$$

Highest frequency is 52 Hz. Nyquist rate is 104 samples/second.

(b) $\mathrm{x}(t) = 4\,\mathrm{sinc}(200t)$

$$\mathrm{X}(f) = (4/200)\mathrm{rect}(f/200)$$

Highest frequency is 100. Nyquist rate is 200 samples/second.

(c) $\mathrm{x}(t) = 45\,\mathrm{sinc}(200t)\sin(20\pi t)$

$$\mathrm{X}(f) = (45/200)\mathrm{rect}(f/200) * (j/2)[\delta(f+10)-\delta(f-10)]$$

$$\mathrm{X}(f) = (j9/80)[\mathrm{rect}((f+10)/200) - \mathrm{rect}((f-10)/200)]$$

Highest frequency is 110 Hz. Nyquist rate is 220 samples/second.

(d) $\mathrm{x}(t) = \begin{cases} 2+\cos(2\pi t) & , |t| \le 1/2 \\ 1 & , |t| > 1/2 \end{cases}$

$$\mathrm{x}(t) = (1+\cos(2\pi t))\mathrm{rect}(t) + 1 \xleftrightarrow{\mathcal{F}} \{\delta(f) + (1/2)[\delta(f-1)+\delta(f+1)]\} * \mathrm{sinc}(f) + \delta(f)$$

Not bandlimited. Nyquist rate is infinite.

(e) $\mathrm{x}(t) = 18\cos(24\pi t) + 35\sin(80\pi t)$

Highest frequency is 40 Hz. Nyquist rate is 80 samples/second.

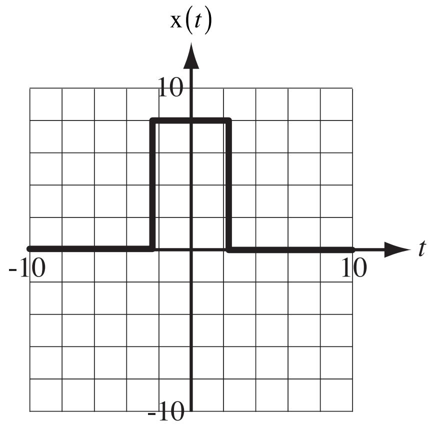
2. Let $\mathrm{x}(t) = 8\,\mathrm{rect}(t/5)$, let an impulse-sampled version be $\mathrm{x}_\delta(t) = \mathrm{x}(t)\delta_{T_s}(t)$ and let $\mathrm{x}_\delta(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}_\delta(f)$.

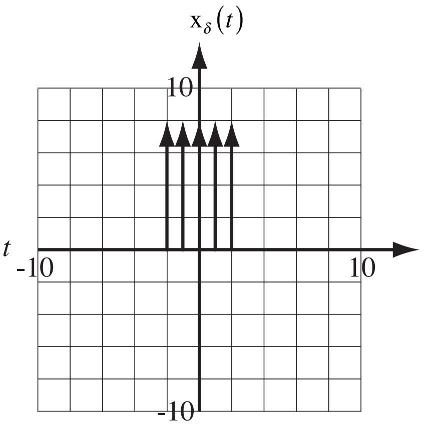
(a) Sketch $\mathrm{x}(t)$ in the axes below. (Put a scale on the vertical axis.)

(b) Sketch $\mathrm{x}_\delta(t) = \mathrm{x}(t)\delta_1(t)$ in the axes below. (Put a scale on the vertical axis.)

(c) (3 pts) The functional behavior of $\mathrm{X}_\delta(f)$ generally depends on T_s but for all values of T_s above some minimum value $T_{s,min}$ $\mathrm{X}_\delta(f)$ is the same. What is the numerical value of $T_{s,min}$? $T_{s,min}$ = ____________

When T_s is greater than 2.5, there is only one impulse inside the rectangle and the impulse-sampled signal always looks the same, one impulse at $t = 0$. Therefore its CTFT is also always the same.





3. Let $x_\delta(t) = K\,\text{tri}(t/4)\delta_4(t-t_0)$ and let $x_\delta(t) \xleftrightarrow{\mathcal{F}} X_\delta(f)$. For $t_0 = 0$, $X_\delta(f) = X_{\delta 0}(f)$ and for $t_0 = 2$, $X_\delta(f) = G(f)X_{\delta 0}(f)$. What is the function $G(f)$?

$G(f)$ = ____

Shifting the periodic impulse to the right by two, puts impulses at $t = \pm 2$, each with half the strength of the original single impulse at $t = 0$. So $G(f) = (1/2)\left(e^{j4\pi f} + e^{-j4\pi f}\right) = \cos(4\pi f)$.

Several students had basically the following analysis:

$$x_\delta(t) = K\,\text{tri}(t/4)\delta_4(t-t_0) \xleftrightarrow{\mathcal{F}} 4K\,\text{sinc}^2(4f) * (1/4)\delta_{1/4}(f)e^{-j2\pi f t_0}$$

$$x_\delta(t) = K\,\text{tri}(t/4)\delta_4(t-t_0) \xleftrightarrow{\mathcal{F}} K\,\text{sinc}^2(4f) * \delta_{1/4}(f)e^{-j2\pi f t_0}$$

For $t_0 = 0$

$$x_\delta(t) = K\,\text{tri}(t/4)\delta_4(t) \xleftrightarrow{\mathcal{F}} K\,\text{sinc}^2(4f) * \delta_{1/4}(f)$$

For $t_0 = 2$

$$x_\delta(t) = K\,\text{tri}(t/4)\delta_4(t-2) \xleftrightarrow{\mathcal{F}} K\,\text{sinc}^2(4f) * \delta_{1/4}(f)e^{-j4\pi f}$$

This analysis is correct up to this point. But then they said that $G(f) = e^{-j4\pi f}$. This is not correct. $\delta_{1/4}(f)$ is multiplied by $e^{-j4\pi f}$ but the whole function $K\,\text{sinc}^2(4f) * \delta_{1/4}(f)$ is not. In order to be able to apply the time shifting property to $K\,\text{tri}(t/4)\delta_4(t) \xleftrightarrow{\mathcal{F}} K\,\text{sinc}^2(4f) * \delta_{1/4}(f)$ the function on the left must be

$$K\,\text{tri}((t-2)/4)\delta_4(t-2) \qquad \text{not} \qquad K\,\text{tri}(t/4)\delta_4(t-2)$$

4. The signal $x(t) = 5\,\text{tri}(10t) * \delta_{0.4}(t)$ is sampled at a rate of 200 samples/second.

(a) If sample #1 occurs at time $t = 0 \times T_s$, and sample #2 occurs at time $t = 1 \times T_s$, etc., what is the numerical value of sample #8?

sample #8 value = ____________

Since sample #1 occurs at $t = 0 = 0 \times T_s$, sample #2 occurs at $t = 1 \times T_s = 5$ ms, etc. Therefore sample #8 occurs at $t = 7 \times T_s = 35$ ms. That time is within the triangle centered at $t = 0$ so the sample value is

$$x(t) = 5\,\text{tri}(10 \times 7T_s) = 5\,\text{tri}(70/200) = 5\,\text{tri}(7/20) = 3.25$$

(b) What is the numerical value of sample #88?

sample #88 value = ____________

This sample occurs at $t = 87T_s = 87/200 = 0.435$ s. The period is 0.4 s. Therefore $x(0.435) = x(0.435 - 0.4n)$ where n is any integer. If we let $n = 1$ then $x(0.435) = x(0.035)$ and

$$x(0.035) = 5\,\text{tri}(0.35) = 5 \times 0.65 = 3.25\,.$$

Solution of ECE 316 Test 2 S10

1. Find the numerical Nyquist rates of these signals. (If a signal is not bandlimited, just write "Infinity" or "∞".)

(a) $\mathrm{x}(t) = 18\cos(24\pi t) + 35\sin(60\pi t)$

Highest frequency is 30 Hz. Nyquist rate is 60 samples/second.

(b) $\mathrm{x}(t) = \cos(24\pi t)\sin(60\pi t)$

$$\mathrm{X}(f) = (1/2)\left[\delta(f-12) + \delta(f+12)\right] * (j/2)\left[\delta(f+30) - \delta(f-30)\right]$$

$$\mathrm{X}(f) = (j/4)\left[\delta(f+18) - \delta(f-42) + \delta(f+52) - \delta(f-18)\right]$$

Highest frequency is 42 Hz. Nyquist rate is 84 samples/second.

(c) $\mathrm{x}(t) = 4\,\mathrm{sinc}(2000t)$

$$\mathrm{X}(f) = (4/2000)\mathrm{rect}(f/2000)$$

Highest frequency is 1000. Nyquist rate is 2000 samples/second.

(d) $\mathrm{x}(t) = 45\,\mathrm{sinc}(2000t)\sin(20\pi t)$

$$\mathrm{X}(f) = (45/2000)\mathrm{rect}(f/2000) * (j/2)\left[\delta(f+10) - \delta(f-10)\right]$$

$$\mathrm{X}(f) = (j9/800)\left[\mathrm{rect}((f+10)/2000) - \mathrm{rect}((f-10)/2000)\right]$$

Highest frequency is 1010 Hz. Nyquist rate is 2020 samples/second.

(e) $\mathrm{x}(t) = \begin{cases} 2 + \cos(2\pi t) & , |t| \le 1/2 \\ 1 & , |t| > 1/2 \end{cases}$

$$\mathrm{x}(t) = (1 + \cos(2\pi t))\mathrm{rect}(t) + 1 \xleftrightarrow{\mathcal{F}} \left\{\delta(f) + (1/2)\left[\delta(f-1) + \delta(f+1)\right]\right\} * \mathrm{sinc}(f) + \delta(f)$$

Not bandlimited. Nyquist rate is infinite.

2. The signal $x(t) = 5\,\text{tri}(10t) * \delta_{0.4}(t)$ is sampled at a rate of 200 samples/second.

(a) If sample #1 occurs at time $t = 0 \times T_s$, and sample #2 occurs at time $t = 1 \times T_s$, etc., what is the numerical value of sample #6?

Since sample #1 occurs at $t = 0 = 0 \times T_s$, sample #2 occurs at $t = 1 \times T_s = 5\text{ ms}$, etc. Therefore sample #6 occurs at $t = 5 \times T_s = 25\text{ ms}$. That time is within the triangle centered at $t = 0$ so the sample value is

$$x(t) = 5\,\text{tri}(10 \times 5T_s) = 5\,\text{tri}(50/200) = 5\,\text{tri}(5/20) = 3.75$$

(b) What is the numerical value of sample #66?

This sample occurs at $t = 65T_s = 65/200 = 0.325\text{ s}$. The period is 0.4 s. Therefore $x(0.325) = x(0.325 - 0.4n)$ where n is any integer. If we let $n = 1$ then $x(0.325) = x(-0.075)$ and

$$x(-0.075) = 5\,\text{tri}(-0.75) = 5 \times 0.25 = 1.25.$$

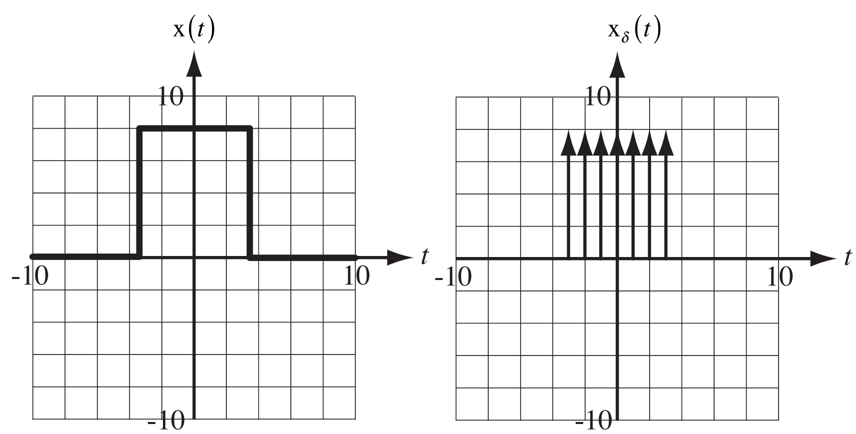
3. Let $x(t) = 8\,\text{rect}(t/7)$, let an impulse-sampled version be $x_\delta(t) = x(t)\delta_{T_s}(t)$ and let $x_\delta(t) \xleftrightarrow{\mathcal{F}} X_\delta(f)$.

(a) Sketch $x(t)$ in the axes below. (Put a scale on the vertical axis.)

(b) Sketch $x_\delta(t) = x(t)\delta_1(t)$ in the axes below. (Put a scale on the vertical axis.)

(c) The functional behavior of $X_\delta(f)$ generally depends on T_s but for all values of T_s above some minimum value $T_{s,min}$ $X_\delta(f)$ is the same. What is the numerical value of $T_{s,min}$? $T_{s,min}$ = ____________

When T_s is greater than 3.5, there is only one impulse inside the rectangle and the impulse-sampled signal always looks the same, one impulse at $t = 0$. Therefore its CTFT is also always the same.



4. Let $\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/8)\delta_8(t-t_0)$ and let $\mathrm{x}_\delta(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}_\delta(f)$. For $t_0 = 0$, $\mathrm{X}_\delta(f) = \mathrm{X}_{\delta 0}(f)$ and for $t_0 = 4$, $\mathrm{X}_\delta(f) = \mathrm{G}(f)\mathrm{X}_{\delta 0}(f)$. What is the function $\mathrm{G}(f)$?

$\mathrm{G}(f)$ = ____________________

Shifting the periodic impulse to the right by 4 puts impulses at $t = \pm 4$, each with half the strength of the original single impulse at $t = 0$. So $\mathrm{G}(f) = (1/2)\left(e^{j8\pi f} + e^{-j4\pi f}\right) = \cos(8\pi f)$.

Several students had basically the following analysis:

$$\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/8)\delta_8(t-t_0) \xleftrightarrow{\mathcal{F}} 8K\,\mathrm{sinc}^2(8f) * (1/8)\delta_{1/8}(f)e^{-j2\pi f t_0}$$

$$\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/8)\delta_8(t-t_0) \xleftrightarrow{\mathcal{F}} K\,\mathrm{sinc}^2(8f) * \delta_{1/8}(f)e^{-j2\pi f t_0}$$

For $t_0 = 0$

$$\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/8)\delta_8(t) \xleftrightarrow{\mathcal{F}} K\,\mathrm{sinc}^2(8f) * \delta_{1/8}(f)$$

For $t_0 = 2$

$$\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/8)\delta_8(t-4) \xleftrightarrow{\mathcal{F}} K\,\mathrm{sinc}^2(8f) * \delta_{1/8}(f)e^{-j8\pi f}$$

This analysis is correct up to this point. But then they said that $\mathrm{G}(f) = e^{-j8\pi f}$. This is not correct. $\delta_{1/8}(f)$ is multiplied by $e^{-j8\pi f}$ but the whole function $K\,\mathrm{sinc}^2(8f) * \delta_{1/8}(f)$ is not. In order to be able to apply the time shifting property to $K\,\mathrm{tri}(t/8)\delta_8(t) \xleftrightarrow{\mathcal{F}} K\,\mathrm{sinc}^2(8f) * \delta_{1/8}(f)$ the function on the left must be

$$K\,\mathrm{tri}((t-4)/8)\delta_8(t-4) \qquad \text{not} \qquad K\,\mathrm{tri}(t/8)\delta_4(t-4)$$

Solution of ECE 316 Test 2 S10

1. The signal $x(t) = 5\,\text{tri}(10t) * \delta_{0.4}(t)$ is sampled at a rate of 200 samples/second.

(a) If sample #1 occurs at time $t = 0 \times T_s$, and sample #2 occurs at time $t = 1 \times T_s$, etc., what is the numerical value of sample #7?

sample #7 value = ____________

Since sample #1 occurs at $t = 0 = 0 \times T_s$, sample #2 occurs at $t = 1 \times T_s = 5\text{ ms}$, etc. Therefore sample #7 occurs at $t = 6 \times T_s = 30\text{ ms}$. That time is within the triangle centered at $t = 0$ so the sample value is

$$x(t) = 5\,\text{tri}(10 \times 6T_s) = 5\,\text{tri}(60/200) = 5\,\text{tri}(3/10) = 3.5$$

(b) What is the numerical value of sample #77?

sample #77 value = ____________

This sample occurs at $t = 76T_s = 76/200 = 0.38\text{ s}$. The period is 0.4 s. Therefore $x(0.38) = x(0.38 - 0.4n)$ where n is any integer. If we let $n = 1$ then $x(0.38) = x(-0.02)$ and

$$x(-0.02) = 5\,\text{tri}(-0.2) = 5 \times 0.8 = 4\ .$$

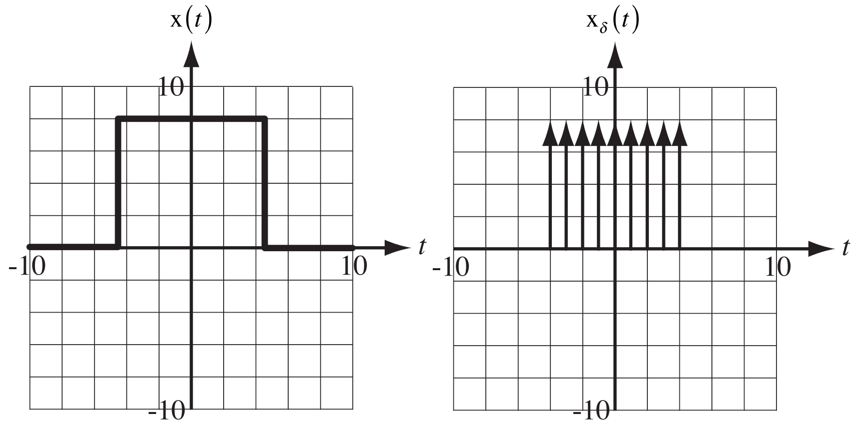
2. Let $\mathrm{x}(t) = 8\,\mathrm{rect}(t/9)$, let an impulse-sampled version be $\mathrm{x}_\delta(t) = \mathrm{x}(t)\delta_{T_s}(t)$ and let $\mathrm{x}_\delta(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}_\delta(f)$.

(a) Sketch $\mathrm{x}(t)$ in the axes below. (Put a scale on the vertical axis.)

(b) Sketch $\mathrm{x}_\delta(t) = \mathrm{x}(t)\delta_1(t)$ in the axes below. (Put a scale on the vertical axis.)

(c) The functional behavior of $\mathrm{X}_\delta(f)$ generally depends on T_s but for all values of T_s above some minimum value $T_{s,min}$ $\mathrm{X}_\delta(f)$ is the same. What is the numerical value of $T_{s,min}$? $T_{s,min}$ = ____________

When T_s is greater than 4.5, there is only one impulse inside the rectangle and the impulse-sampled signal always looks the same, one impulse at $t = 0$. Therefore its CTFT is also always the same.



3. Let $\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/6)\delta_6(t - t_0)$ and let $\mathrm{x}_\delta(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}_\delta(f)$. For $t_0 = 0$, $\mathrm{X}_\delta(f) = \mathrm{X}_{\delta 0}(f)$ and for $t_0 = 3$, $\mathrm{X}_\delta(f) = \mathrm{G}(f)\mathrm{X}_{\delta 0}(f)$. What is the function $\mathrm{G}(f)$?

$\mathrm{G}(f)$ = ____

Shifting the periodic impulse to the right by two puts impulses at $t = \pm 3$, each with half the strength of the original single impulse at $t = 0$. So $\mathrm{G}(f) = (1/2)\left(e^{j6\pi f} + e^{-j6\pi f}\right) = \cos(6\pi f)$.

Several students had basically the following analysis:

$$\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/6)\delta_6(t - t_0) \xleftrightarrow{\mathcal{F}} 6K\,\mathrm{sinc}^2(6f) * (1/6)\delta_{1/6}(f)e^{-j2\pi f t_0}$$

$$\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/6)\delta_6(t - t_0) \xleftrightarrow{\mathcal{F}} K\,\mathrm{sinc}^2(6f) * \delta_{1/6}(f)e^{-j2\pi f t_0}$$

For $t_0 = 0$

$$\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/6)\delta_6(t) \xleftrightarrow{\mathcal{F}} K\,\mathrm{sinc}^2(6f) * \delta_{1/6}(f)$$

For $t_0 = 2$

$$\mathrm{x}_\delta(t) = K\,\mathrm{tri}(t/6)\delta_6(t - 4) \xleftrightarrow{\mathcal{F}} K\,\mathrm{sinc}^2(6f) * \delta_{1/6}(f)e^{-j6\pi f}$$

This analysis is correct up to this point. But then they said that $\mathrm{G}(f) = e^{-j6\pi f}$. This is not correct. $\delta_{1/6}(f)$ is multiplied by $e^{-j6\pi f}$ but the whole function $K\,\mathrm{sinc}^2(6f) * \delta_{1/6}(f)$ is not. In order to be able to apply the time shifting property to $K\,\mathrm{tri}(t/6)\delta_6(t) \xleftrightarrow{\mathcal{F}} K\,\mathrm{sinc}^2(6f) * \delta_{1/6}(f)$ the function on the left must be

$$K\,\mathrm{tri}\left(\left(t-3\right)/6\right)\delta_4\left(t-3\right) \qquad \text{not} \qquad K\,\mathrm{tri}\left(t/6\right)\delta_4\left(t-3\right)$$

4. Find the numerical Nyquist rates of these signals. (If a signal is not bandlimited, just write "Infinity" or "∞".)

(a) $x(t) = 18\cos(240\pi t) + 35\sin(800\pi t)$

Highest frequency is 400 Hz. Nyquist rate is 800 samples/second.

(b) $x(t) = \cos(240\pi t)\sin(800\pi t)$

$$X(f) = (1/2)[\delta(f-120) + \delta(f+120)] * (j/2)[\delta(f+400) - \delta(f-400)]$$

$$X(f) = (j/4)[\delta(f+280) - \delta(f-520) + \delta(f+520) - \delta(f-280)]$$

Highest frequency is 520 Hz. Nyquist rate is 1040 samples/second.

(c) $x(t) = 4\,\mathrm{sinc}(20t)$

$X(f) = (4/20)\mathrm{rect}(f/20)$

Highest frequency is 10. Nyquist rate is 20 samples/second.

(d) $x(t) = 45\,\mathrm{sinc}(20t)\sin(2\pi t)$

$$X(f) = (45/20)\mathrm{rect}(f/20) * (j/2)[\delta(f+1) - \delta(f-1)]$$

$$X(f) = (j9/8)[\mathrm{rect}((f+1)/20) - \mathrm{rect}((f-1)/20)]$$

Highest frequency is 11 Hz. Nyquist rate is 22 samples/second.

(e) $x(t) = \begin{cases} 2 + \cos(2\pi t) & , |t| \le 1/2 \\ 1 & , |t| > 1/2 \end{cases}$

$x(t) = (1 + \cos(2\pi t))\mathrm{rect}(t) + 1 \xleftrightarrow{\mathcal{F}} \{\delta(f) + (1/2)[\delta(f-1) + \delta(f+1)]\} * \mathrm{sinc}(f) + \delta(f)$

Not bandlimited. Nyquist rate is infinite.