Solution of EECS 316 Test 1 Su10

1. Find the numerical values of the constants and, for the forward transforms, the region of convergence.

(a) $\delta(t-1)-2\delta(t-3) \overset{\mathscr{L}}{\longleftrightarrow} e^{-s}\left(A-Be^{cs}\right)$

$\delta(t-1)-2\delta(t-3) \overset{\mathscr{L}}{\longleftrightarrow} e^{-s}\left(1-2e^{-2s}\right)$, ROC = All s

(b) $\cos(20\pi t)\,\mathrm{u}(t) * \left[\mathrm{u}(t)-\mathrm{u}(t-1)\right] \overset{\mathscr{L}}{\longleftrightarrow} \dfrac{A+Be^{cs}}{s^2+D}$

$\cos(20\pi t)\,\mathrm{u}(t) * \left[\mathrm{u}(t)-\mathrm{u}(t-1)\right] \overset{\mathscr{L}}{\longleftrightarrow} \dfrac{s}{s^2+(20\pi)^2}\left[\dfrac{1-e^{-s}}{s}\right] = \dfrac{1-e^{-s}}{s^2+(20\pi)^2}$, ROC = $\sigma > 0$

(c) $Ae^{bt}\sin(Ct)\,\mathrm{u}(dt) \overset{\mathscr{L}}{\longleftrightarrow} \dfrac{15}{(s-3)^2+100}$, $\sigma < 3$

$-1.5e^{3t}\sin(10t)\,\mathrm{u}(-t) \overset{\mathscr{L}}{\longleftrightarrow} \dfrac{15}{(s-3)^2+100}$, $\sigma < 3$

(d) $4n\,\mathrm{u}[n+1] \overset{\mathscr{Z}}{\longleftrightarrow} Az^2\dfrac{z-b}{(z-c)^2}$

(Hint: Express the time-domain function as the sum of a causal function and an anti-causal function, combine the z-transform results over a common denominator and simplify.)

$4n\,\mathrm{u}[n+1] = 4\left(-\delta[n+1]+n\,\mathrm{u}[n]\right) \overset{\mathscr{Z}}{\longleftrightarrow} 4\left[-z+\dfrac{z}{(z-1)^2}\right] = -4z^2\dfrac{z-2}{(z-1)^2}$, ROC = $|z| > 1$

(e) $4n\,\mathrm{u}[n-1] \overset{\mathscr{Z}}{\longleftrightarrow} \dfrac{Az}{(z-a)^2}$

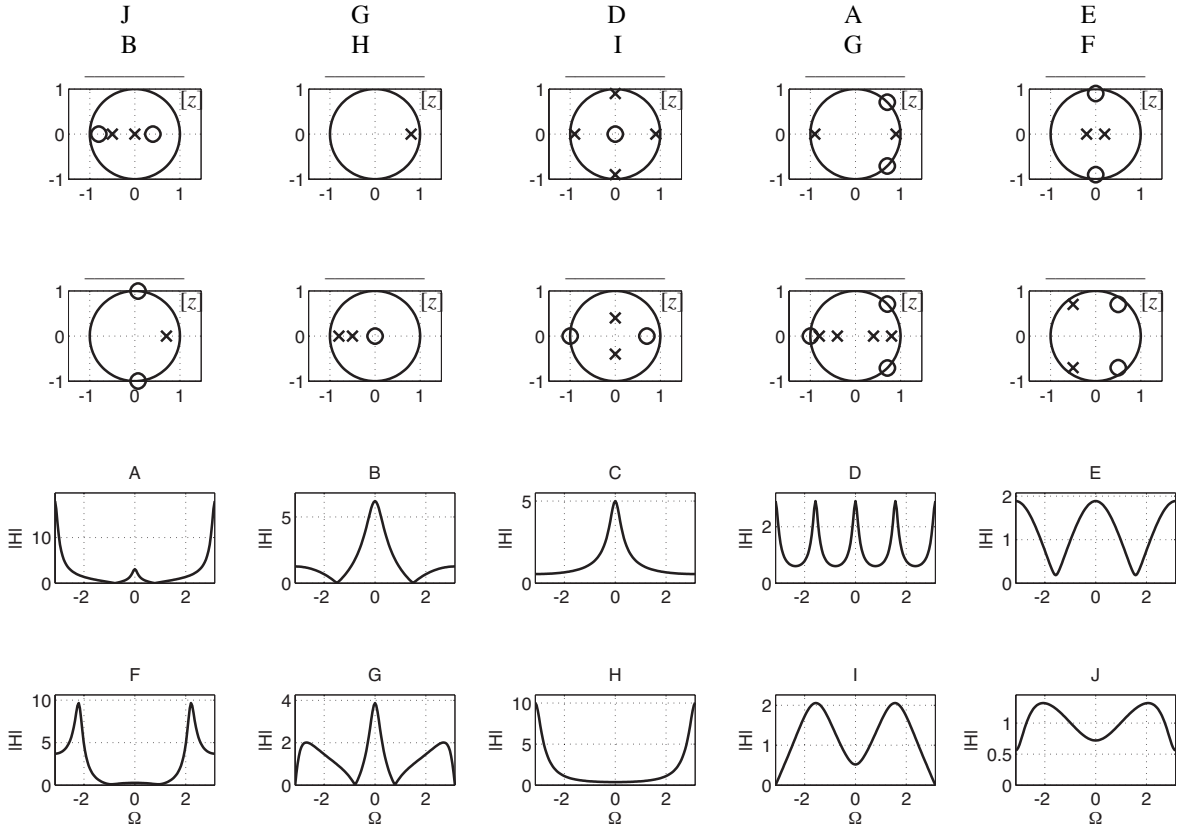
$4n\,\mathrm{u}[n-1] = 4\left\{(n-1)\,\mathrm{u}[n-1]+\mathrm{u}[n-1]\right\} \overset{\mathscr{Z}}{\longleftrightarrow} 4\left(\dfrac{1}{(z-1)^2}+\dfrac{1}{z-1}\right) = \dfrac{4z}{(z-1)^2}$, ROC = $|z| > 1$

(f) $Ab^n\,\mathrm{u}[-n-1]+Cd^n\,\mathrm{u}[n] \overset{\mathscr{Z}}{\longleftrightarrow} \dfrac{z^2}{(z+0.5)(z-0.2)}$, $0.2 < |z| < 0.5$

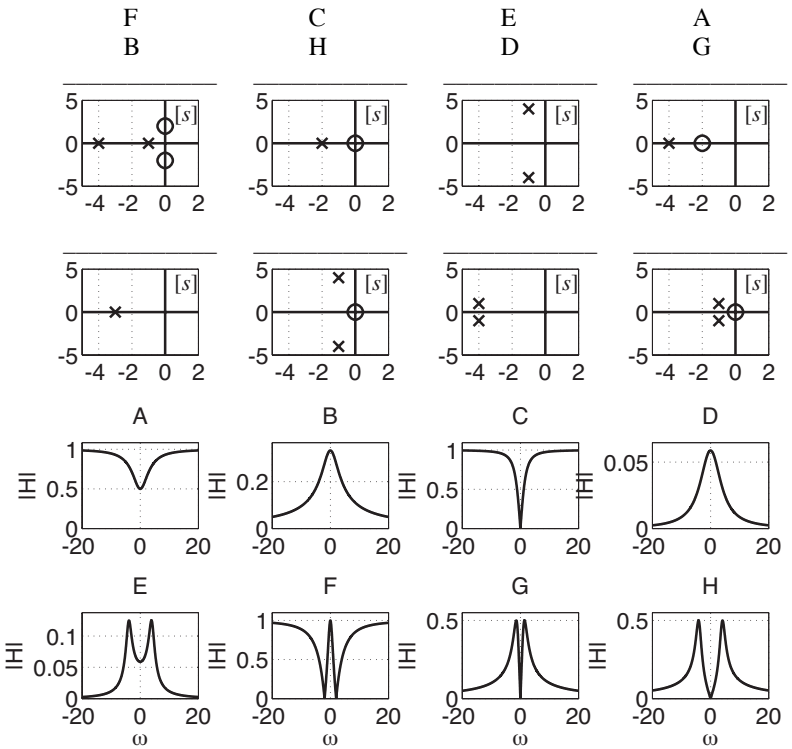
$\dfrac{z^2}{(z+0.5)(z-0.2)} = \dfrac{5z/7}{z+0.5}+\dfrac{2z/7}{z-0.2}$

$-(5/7)(-0.5)^n\,\mathrm{u}[-n-1]+(2/7)(0.2)^n\,\mathrm{u}[n] \overset{\mathscr{Z}}{\longleftrightarrow} \dfrac{z^2}{(z+0.5)(z-0.2)}$, $0.2 < z < 0.5$

2. (20 pts) Match pole-zero diagrams to frequency responses by writing the letter designation of the magnitude frequency response that matches each pole-zero diagram in the blank space above that pole-zero diagram.



3. Match pole-zero diagrams to frequency responses by writing the letter designation of the magnitude frequency response that matches each pole-zero diagram in the blank space above that pole-zero diagram.



Solution of EECS 316 Test 1 Su10

1. Find the numerical values of the constants and, for the forward transforms, the region of convergence.

(a) $2\delta(t-1)-\delta(t-4)\xleftrightarrow{\mathcal{L}} e^{-s}\left(A-Be^{cs}\right)$

$2\delta(t-1)-\delta(t-4)\xleftrightarrow{\mathcal{L}} e^{-s}\left(2-e^{-3s}\right)$, ROC = All s

(b) $\cos(20t)\,\mathrm{u}(t)*\left[\mathrm{u}(t)-\mathrm{u}(t-2)\right]\xleftrightarrow{\mathcal{L}} \dfrac{A+Be^{cs}}{s^2+D}$

$\cos(20t)\,\mathrm{u}(t)*\left[\mathrm{u}(t)-\mathrm{u}(t-2)\right]\xleftrightarrow{\mathcal{L}} \dfrac{s}{s^2+(20)^2}\left[\dfrac{1-e^{-2s}}{s}\right]=\dfrac{1-e^{-2s}}{s^2+(20)^2}$, ROC = $\sigma>0$

(c) $Ae^{bt}\sin(Ct)\,\mathrm{u}(dt)\xleftrightarrow{\mathcal{L}} \dfrac{75}{(s-6)^2+144}$, $\sigma<6$

$-6.25e^{6t}\sin(12t)\,\mathrm{u}(-t)\xleftrightarrow{\mathcal{L}} \dfrac{75}{(s-6)^2+144}$, $\sigma<6$

(d) $9n\,\mathrm{u}[n+1]\xleftrightarrow{\mathcal{Z}} Az^2\dfrac{z-b}{(z-c)^2}$

(Hint: Express the time-domain function as the sum of a causal function and an anti-causal function, combine the z-transform results over a common denominator and simplify.)

$9n\,\mathrm{u}[n+1]=9\left(-\delta[n+1]+n\,\mathrm{u}[n]\right)\xleftrightarrow{\mathcal{Z}} 9\left[-z+\dfrac{z}{(z-1)^2}\right]=-9z^2\dfrac{z-2}{(z-1)^2}$, ROC = $|z|>1$

(e) $4n\,\mathrm{u}[n-1]\xleftrightarrow{\mathcal{Z}} \dfrac{Az}{(z-a)^2}$

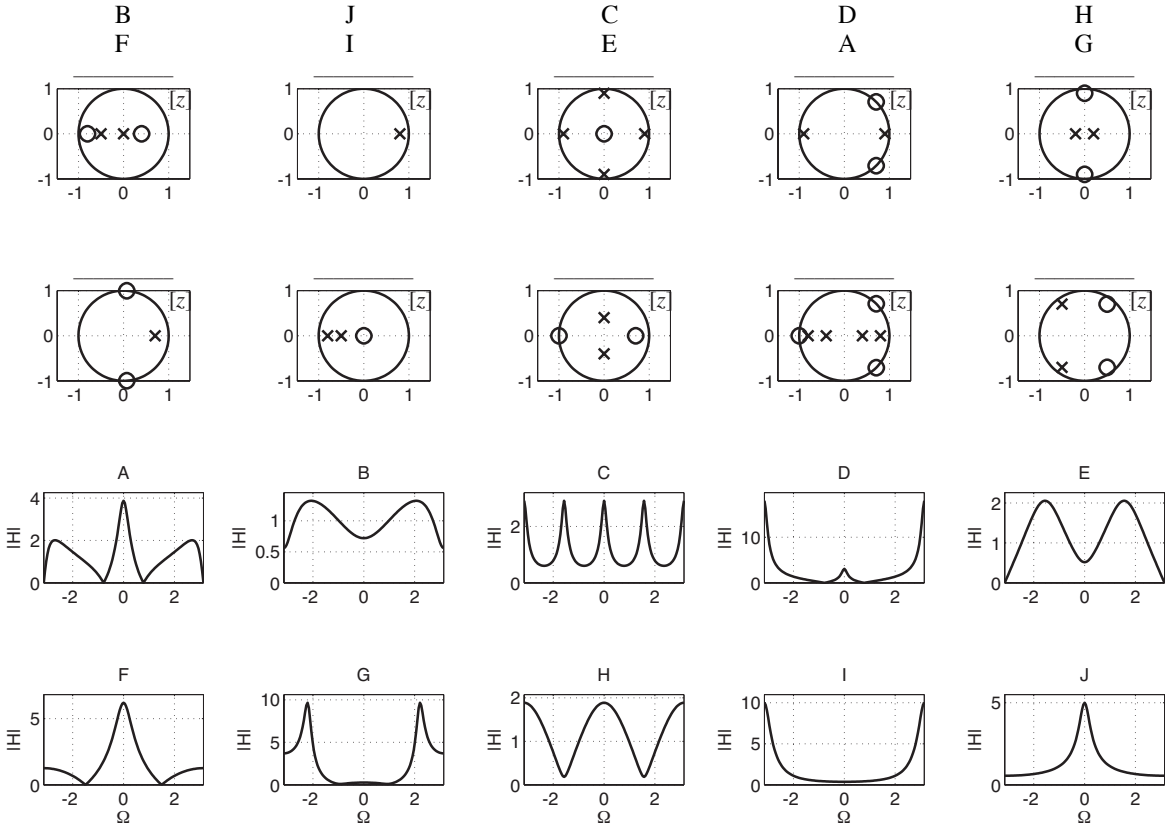
$4n\,\mathrm{u}[n-1]=4\left\{(n-1)\mathrm{u}[n-1]+\mathrm{u}[n-1]\right\}\xleftrightarrow{\mathcal{Z}} 4\left(\dfrac{1}{(z-1)^2}+\dfrac{1}{z-1}\right)=\dfrac{4z}{(z-1)^2}$, ROC = $|z|>1$

(f) $Ab^n\,\mathrm{u}[-n-1]+Cd^n\,\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}} \dfrac{z^2}{(z+0.5)(z-0.2)}$, $0.2<|z|<0.5$

$\dfrac{z^2}{(z+0.5)(z-0.2)}=\dfrac{5z/7}{z+0.5}+\dfrac{2z/7}{z-0.2}$

$-(5/7)(-0.5)^n\,\mathrm{u}[-n-1]+(2/7)(0.2)^n\,\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}} \dfrac{z^2}{(z+0.5)(z-0.2)}$, $0.2<z<0.5$

2. (20 pts) Match pole-zero diagrams to frequency responses by writing the letter designation of the magnitude frequency response that matches each pole-zero diagram in the blank space above that pole-zero diagram.



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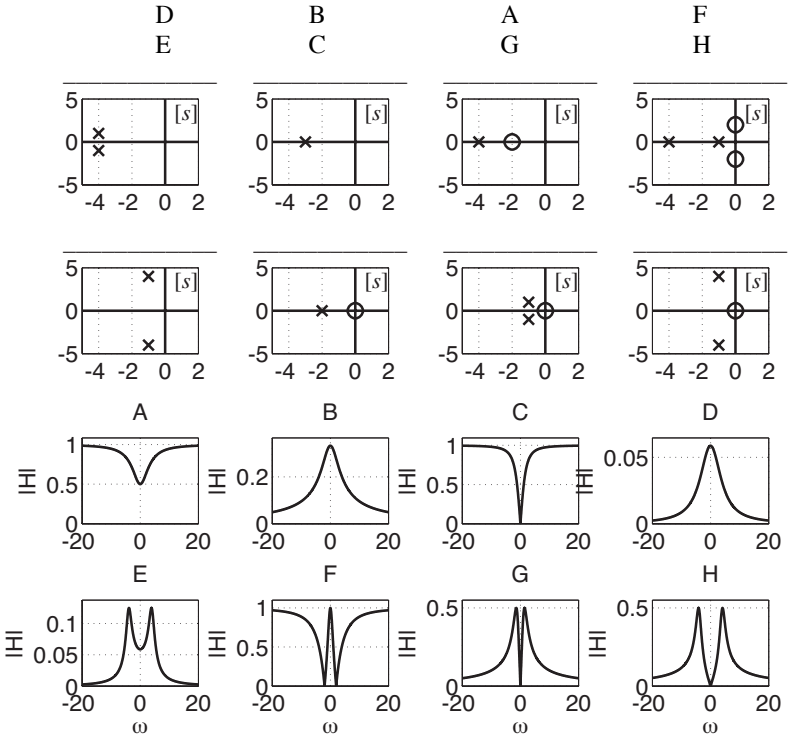

Solution of EECS 316 Test 1 Su10

1. Find the numerical values of the constants and, for the forward transforms, the region of convergence.

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$4\delta(t-1)-3\delta(t-4)\overset{\mathscr{L}}{\longleftrightarrow}e^{-s}\left(4-3e^{-3s}\right)$, ROC = All s

(b) $\cos(50t)\,\mathrm{u}(t)*\left[\mathrm{u}(t)-\mathrm{u}(t-2)\right]\overset{\mathscr{L}}{\longleftrightarrow}\dfrac{A+Be^{cs}}{s^2+D}$

$\cos(50t)\,\mathrm{u}(t)*\left[\mathrm{u}(t)-\mathrm{u}(t-2)\right]\overset{\mathscr{L}}{\longleftrightarrow}\dfrac{s}{s^2+(50)^2}\left[\dfrac{1-e^{-2s}}{s}\right]=\dfrac{1-e^{-2s}}{s^2+(50)^2}$, ROC = $\sigma>0$

(c) $Ae^{bt}\sin(Ct)\,\mathrm{u}(dt)\overset{\mathscr{L}}{\longleftrightarrow}\dfrac{75}{(s-2)^2+49}$, $\sigma<2$

$-10.714e^{2t}\sin(7t)\,\mathrm{u}(-t)\overset{\mathscr{L}}{\longleftrightarrow}\dfrac{75}{(s-2)^2+49}$, $\sigma<2$

(d) $7n\,\mathrm{u}[n+1]\overset{\mathscr{Z}}{\longleftrightarrow}Az^2\dfrac{z-b}{(z-c)^2}$

(Hint: Express the time-domain function as the sum of a causal function and an anti-causal function, combine the z-transform results over a common denominator and simplify.)

$7n\,\mathrm{u}[n+1]=7\left(-\delta[n+1]+n\,\mathrm{u}[n]\right)\overset{\mathscr{Z}}{\longleftrightarrow}7\left[-z+\dfrac{z}{(z-1)^2}\right]=-7z^2\dfrac{z-2}{(z-1)^2}$, ROC = $|z|>1$

(e) $7n\,\mathrm{u}[n-1]\overset{\mathscr{Z}}{\longleftrightarrow}\dfrac{Az}{(z-a)^2}$

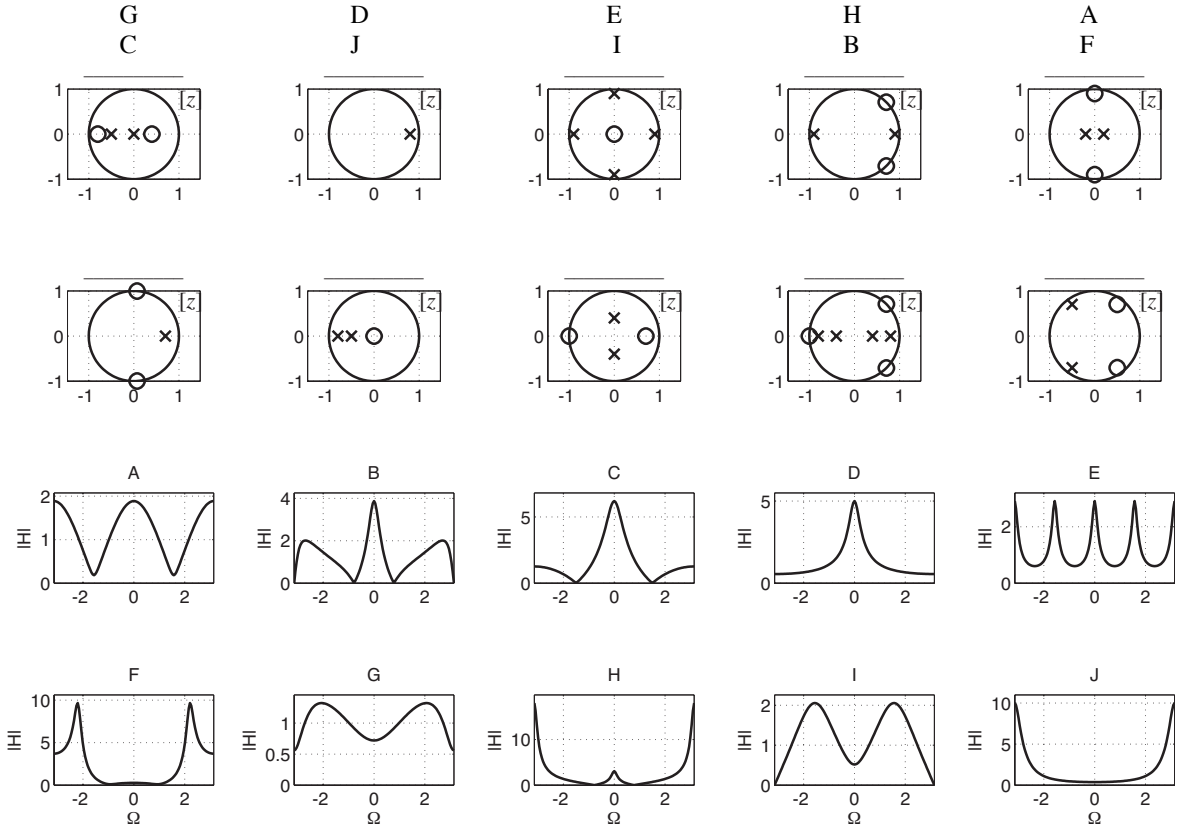
$7n\,\mathrm{u}[n-1]=7\left\{(n-1)\mathrm{u}[n-1]+\mathrm{u}[n-1]\right\}\overset{\mathscr{Z}}{\longleftrightarrow}7\left(\dfrac{1}{(z-1)^2}+\dfrac{1}{z-1}\right)=\dfrac{7z}{(z-1)^2}$, ROC = $|z|>1$

(f) $Ab^n\,\mathrm{u}[-n-1]+Cd^n\,\mathrm{u}[n]\overset{\mathscr{Z}}{\longleftrightarrow}\dfrac{z^2}{(z+0.7)(z-0.1)}$, $0.1<|z|<0.7$

$\dfrac{z^2}{(z+0.7)(z-0.1)}=\dfrac{7z/8}{z+0.7}+\dfrac{z/8}{z-0.1}$

$-(7/8)(-0.7)^n\,\mathrm{u}[-n-1]+(1/8)(0.1)^n\,\mathrm{u}[n]\overset{\mathscr{Z}}{\longleftrightarrow}\dfrac{z^2}{(z+0.7)(z-0.1)}$, $0.1<z<0.7$

2. (20 pts) Match pole-zero diagrams to frequency responses by writing the letter designation of the magnitude frequency response that matches each pole-zero diagram in the blank space above that pole-zero diagram.



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