# Solution of EECS 316 Test 2 Su10

1. A signal $x(t) = \cos(200\pi t)$ is impulse sampled to form $x_\delta(t) = \cos(200\pi t)\delta_{T_s}(t)$ , $T_s = 1/f_s$. For each sampling rate $f_s$ below find the first three positive numerical values of frequency *f* at which the CTFT of $x_\delta(t)$ <u>is not zero</u>.

   (a) $f_s = 150$

   The impulses in the CTFT will occur at $\pm 100 \pm 150n$. Those values will be $50, 100, 250, 200, \ldots$ . So the answers are 50, 100, 200.

   (b) $f_s = 40$

   The impulses in the CTFT will occur at $\pm 100 \pm 40n$. Those values will be $20, 60, 100, 140, 180, 220, 260, \ldots$. So the answers are 20, 60, 100.

2. A signal $x(t) = 12\,\text{sinc}(30t)$ is impulse sampled to form $x_\delta(t) = 12\,\text{sinc}(30t)\delta_{T_s}(t)$ , $T_s = 1/f_s$. For each sampling rate $f_s$ below find all the numerical frequency ranges between 0 and 200 Hz at which the CTFT of $x_\delta(t)$ <u>is zero</u>.

   (a) $f_s = 150$

   The CTFT of the original signal is $X(f) = (2/5)\text{rect}(f/30)$. The CTFT of the impulse-sampled signal is then $X(f) = 150(2/5)\text{rect}(f/30) * \delta_{150}(f)$. The ranges at which the CTFT is NOT zero are

   $$-15 < f < 15 \;,\; 135 < f < 165 \;,\; 285 < f < 315 \;,\; \ldots$$

   So the ranges of frequencies at which the CTFT IS zero are $15 < f < 135$ , $165 < f < 200$

   (b) $f_s = 40$

   CTFT of the impulse-sampled signal is then $X(f) = 40(2/5)\text{rect}(f/30) * \delta_{40}(f)$. The ranges at which the CTFT is NOT zero are

   $$-15 < f < 15 \;,\; 25 < f < 55 \;,\; 65 < f < 95 \;,\; 105 < f < 135 \;,\; 145 < f < 175 \;,\; 185 < f < 215 \;,\; \ldots$$

   So the ranges of frequencies at which the CTFT IS zero are

   $$15 < f < 25 \;,\; 55 < f < 65 \;,\; 95 < f < 105 \;,\; 135 < f < 145 \;,\; 175 < f < 185 \;.$$

3. Find the numerical Nyquist rates for the following signals. If a signal is not bandlimited, just write "infinity" or $\infty$.

(a) $\mathrm{x}(t) = 11\,\mathrm{tri}(3t) * \delta_1(t)$

$$\mathrm{X}(f) = (11/3)\mathrm{sinc}^2(f/3)\delta_1(f) \qquad \text{Not Bandlimited}$$

(b) $\mathrm{x}(t) = 11\,\mathrm{sinc}(3t)\delta_1(t) = \delta(t)$

$$\mathrm{X}(f) = (11/3)\mathrm{rect}(f/3) * \delta_1(f) = 11 \qquad \text{Not Bandlimited}$$

(c) $\mathrm{x}(t) = 8\sin(35t)\cos(20\pi t)$

$$\mathrm{X}(f) = (j2)\left[\delta(f+17.5/\pi) - \delta(f-17.5/\pi)\right] * \left[\delta(f-10) + \delta(f+10)\right]$$
$$\mathrm{X}(f) = (j2)\left[\delta(f+17.5/\pi-10) + \delta(f+17.5/\pi+10) - \delta(f-17.5/\pi-10) - \delta(f-17.5/\pi+10)\right]$$

Nyquist rate is $f_{NYQ} = 2 \times (17.5/\pi + 10) = 31.141$

(d) $\mathrm{x}(t) = \left[\mathrm{u}(t) - \mathrm{u}(t-3)\right]\sin(28\pi t)$ Time-limited, therefore not Bandlimited

(e) $\mathrm{x}(t) = 4\cos(50\pi t) - 6\sin(78\pi t)$ Highest frequency is 39 Hz. $f_{NYQ} = 78$

4. A discrete-time system has a transfer function $\mathrm{H}(z) = \dfrac{z^2 - 1}{z^2 + 0.95}$.

(a) At what numerical radian frequency or frequencies in the range $-\pi \le \Omega < \pi$ is the magnitude of this system's frequency response a minimum?

Minimum occurs at $z = \pm 1 \Rightarrow \Omega = -\pi, 0$

(b) At what numerical radian frequency or frequencies in the range $-\pi \le \Omega < \pi$ is the magnitude of this system's frequency response a maximum?

Maximum occurs at nearest approach to poles at $z = \pm j0.975 \Rightarrow \Omega = \pm \pi/2$

(c) What is the numerical magnitude <u>in dB</u> of the frequency response at $\Omega = 1.6$.

$$\mathrm{H}(e^{j\Omega}) = \frac{e^{j2\Omega} - 1}{e^{j2\Omega} + 0.95} \Rightarrow \mathrm{H}(e^{j2}) = \frac{e^{j3.2} - 1}{e^{j3.2} + 0.95} = 26.387e^{-j0.8504} \Rightarrow \left|\mathrm{H}(e^{j\Omega})\right|_{\mathrm{dB}} = 28.428 \text{ dB}$$

5. The frequency response of a continuous-time LTI system is $\mathrm{H}(j\omega) = \dfrac{\mathrm{V}_o(j\omega)}{\mathrm{V}_i(j\omega)}$. The magnitude Bode diagram for this system is displayed below. (The dashed lines are asymptotes.)

(a) List the numerical locations in radians/second of all finite poles.

Poles at 10 and 8000

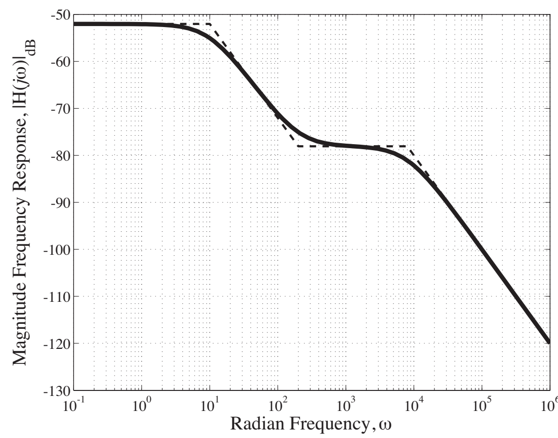
(b) List the numerical locations in radians/second of all finite zeros.

Zero at 200

(c) A sinusoidal signal $\mathrm{x}(t) = 3\sin(1000t)$ volts is applied to the system. What is the numerical amplitude of the sinusoidal response in volts?

Frequency response magnitude at $\omega = 1000$ is about -77 dB. So the output signal response amplitude $A$ can be found from

$$20\log_{10}(A/3) = -77 \Rightarrow A = 3\times 10^{-77/20} = 0.0004238$$



# Solution of EECS 316 Test 2 Su10

1. A signal $x(t)=\cos(220\pi t)$ is impulse sampled to form $x_\delta(t)=\cos(220\pi t)\delta_{T_s}(t)$ , $T_s=1/f_s$. For each sampling rate $f_s$ below find the first three positive numerical values of frequency $f$ at which the CTFT of $x_\delta(t)$ <u>is not zero</u>.

   (a) $f_s=150$

   The impulses in the CTFT will occur at $\pm 110 \pm 150n$. Those values will be $40,110,260,190,...$ . So the answers are 40, 110, 190.

   (b) $f_s=40$

   The impulses in the CTFT will occur at $\pm 110 \pm 40n$. Those values will be $10,30,50,70,110,150,190,230,270,...$. So the answers are 10, 30, 50.

2. A signal $x(t)=12\,\mathrm{sinc}(34t)$ is impulse sampled to form $x_\delta(t)=12\,\mathrm{sinc}(34t)\delta_{T_s}(t)$ , $T_s=1/f_s$. For each sampling rate $f_s$ below find all the numerical frequency ranges between 0 and 200 Hz at which the CTFT of $x_\delta(t)$ <u>is zero</u>.

   (a) $f_s=150$

   The CTFT of the original signal is $X(f)=(6/17)\mathrm{rect}(f/34)$. The CTFT of the impulse-sampled signal is then $X(f)=150(6/17)\mathrm{rect}(f/34)*\delta_{150}(f)$. The ranges at which the CTFT is NOT zero are

$$-17<f<17\ ,\ 133<f<167\ ,\ 283<f<317\ ,\ ...$$

   So the ranges of frequencies at which the CTFT IS zero are $17<f<133$ , $167<f<200$

   (b) $f_s=40$

   CTFT of the impulse-sampled signal is then $X(f)=40(6/17)\mathrm{rect}(f/34)*\delta_{40}(f)$. The ranges at which the CTFT is NOT zero are

$$-17<f<17\ ,\ 23<f<57\ ,\ 63<f<97\ ,\ 103<f<137\ ,\ 143<f<177\ ,\ 183<f<217\ ,\ ...$$

   So the ranges of frequencies at which the CTFT IS zero are

$$17<f<23\ ,\ 57<f<63\ ,\ 97<f<103\ ,\ 137<f<143\ ,\ 177<f<183\ .$$

3. Find the numerical Nyquist rates for the following signals. If a signal is not bandlimited, just write "infinity" or $\infty$.

(a) $\mathrm{x}(t)=11\,\mathrm{tri}(3t)*\delta_1(t)$

$$\mathrm{X}(f)=(11/3)\mathrm{sinc}^2(f/3)\delta_1(f) \qquad \text{Not Bandlimited}$$

(b) $\mathrm{x}(t)=11\,\mathrm{sinc}(3t)*\delta_1(t)$

$$\mathrm{X}(f)=(11/3)\mathrm{rect}(f/3)\delta_1(f) \qquad \text{Highest frequency is 1 Hz.} \quad f_{NYQ}=2$$

(c) $\mathrm{x}(t)=8\sin(35t)\cos(10\pi t)$

$$\mathrm{X}(f)=(j2)\left[\delta(f+17.5/\pi)-\delta(f-17.5/\pi)\right]*\left[\delta(f-5)+\delta(f+5)\right]$$
$$\mathrm{X}(f)=(j2)\left[\delta(f+17.5/\pi-5)+\delta(f+17.5/\pi+5)-\delta(f-17.5/\pi-5)-\delta(f-17.5/\pi+5)\right]$$

Nyquist rate is $f_{NYQ}=2\times(17.5/\pi+5)=21.141$

(d) $\mathrm{x}(t)=\left[\mathrm{u}(t)-\mathrm{u}(t-3)\right]\sin(28\pi t)$ Time-limited, therefore not Bandlimited

(e) $\mathrm{x}(t)=4\cos(50\pi t)-6\sin(88\pi t)$ Highest frequency is 44 Hz. $f_{NYQ}=88$

4. A discrete-time system has a transfer function $\mathrm{H}(z)=\dfrac{z^2-1}{z^2+0.95}$.

(a) At what numerical radian frequency or frequencies in the range $-\pi\le\Omega<\pi$ is the magnitude of this system's frequency response a minimum?

Minimum occurs at $z=\pm1\Rightarrow\Omega=-\pi,0$

(b) At what numerical radian frequency or frequencies in the range $-\pi\le\Omega<\pi$ is the magnitude of this system's frequency response a maximum?

Maximum occurs at nearest approach to poles at $z=\pm j0.975\Rightarrow\Omega=\pm\pi/2$

(c) What is the numerical magnitude <u>in dB</u> of the frequency response at $\Omega=1.4$.

$$\mathrm{H}(e^{j\Omega})=\frac{e^{j2\Omega}-1}{e^{j2\Omega}+0.95}\Rightarrow\mathrm{H}(e^{j2})=\frac{e^{j2.8}-1}{e^{j2.8}+0.95}=5.882e^{j1.4232}\Rightarrow\left|\mathrm{H}(e^{j\Omega})\right|_{\mathrm{dB}}=15.39\text{ dB}$$

5. The frequency response of a continuous-time LTI system is $\mathrm{H}(j\omega)=\frac{\mathrm{V}_o(j\omega)}{\mathrm{V}_i(j\omega)}$. The magnitude Bode diagram for this system is displayed below. (The dashed lines are asymptotes.)

(a) List the numerical locations in radians/second of all finite poles.

Poles at 30 and 7000

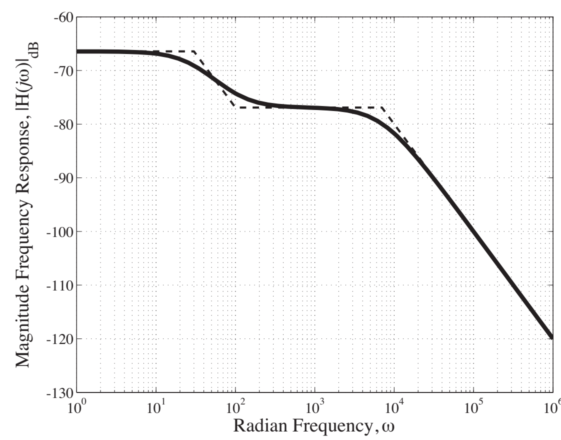
(b) (1 pt per correct zero location) List the numerical locations in radians/second of all finite zeros.

Zero at 100

(c) A sinusoidal signal $\mathrm{x}(t)=3\sin(3t)$ volts is applied to the system. What is the numerical amplitude of the sinusoidal response in volts?

Frequency response magnitude at $\omega=3$ is about -66 dB. So the output signal response amplitude $A$ can be found from

$$20\log_{10}(A/3)=-66\Rightarrow A=3\times10^{-66/20}=0.0015$$



# Solution of EECS 316 Test 2 Su10

1. A signal $x(t) = \cos(160\pi t)$ is impulse sampled to form $x_\delta(t) = \cos(160\pi t)\delta_{T_s}(t)$ , $T_s = 1/f_s$. For each sampling rate $f_s$ below find the first three positive numerical values of frequency $f$ at which the CTFT of $x_\delta(t)$ <u>is not zero</u>.

(a) $f_s = 150$

The impulses in the CTFT will occur at $\pm 80 \pm 150n$. Those values will be $70, 80, 220, 230, \dots$. So the answers are 70, 80, 220.

(b) $f_s = 40$

The impulses in the CTFT will occur at $\pm 80 \pm 40n$. Those values will be $40, 80, 120, 160, 200, 240, \dots$. So the answers are 40, 80, 120.

2. A signal $x(t) = 12\,\mathrm{sinc}(26t)$ is impulse sampled to form $x_\delta(t) = 12\,\mathrm{sinc}(26t)\delta_{T_s}(t)$ , $T_s = 1/f_s$. For each sampling rate $f_s$ below find all the numerical frequency ranges between 0 and 200 Hz at which the CTFT of $x_\delta(t)$ <u>is zero</u>.

(a) $f_s = 150$

The CTFT of the original signal is $X(f) = (6/13)\mathrm{rect}(f/26)$. The CTFT of the impulse-sampled signal is then $X(f) = 150(6/13)\mathrm{rect}(f/26) * \delta_{150}(f)$. The ranges at which the CTFT is NOT zero are

$$-13 < f < 13\ ,\ 137 < f < 163\ ,\ 287 < f < 313\ ,\ \dots$$

So the ranges of frequencies at which the CTFT IS zero are $13 < f < 137$ , $163 < f < 200$

(b) $f_s = 40$

CTFT of the impulse-sampled signal is then $X(f) = 40(6/13)\mathrm{rect}(f/26) * \delta_{40}(f)$. The ranges at which the CTFT is NOT zero are

$$-13 < f < 13\ ,\ 27 < f < 53\ ,\ 67 < f < 93\ ,\ 107 < f < 133\ ,\ 147 < f < 173\ ,\ 187 < f < 213\ ,\ \dots$$

So the ranges of frequencies at which the CTFT IS zero are

$$13 < f < 27\ ,\ 53 < f < 67\ ,\ 93 < f < 107\ ,\ 133 < f < 147\ ,\ 173 < f < 187\ .$$

3. Find the numerical Nyquist rates for the following signals. If a signal is not bandlimited, just write "infinity" or $\infty$.

(a) $x(t) = 11\,\text{tri}(3t) * \delta_1(t)$

$$X(f) = (11/3)\text{sinc}^2(f/3)\delta_1(f) \qquad \text{Not Bandlimited}$$

(b) $x(t) = 11\,\text{sinc}(3t)\delta_1(t) = \delta(t)$

$$X(f) = (11/3)\text{rect}(f/3) * \delta_1(f) = 11 \qquad \text{Not Bandlimited}$$

(c) $x(t) = 8\sin(35t)\cos(30\pi t)$

$$X(f) = (j2)\left[\delta(f + 17.5/\pi) - \delta(f - 17.5/\pi)\right] * \left[\delta(f - 15) + \delta(f + 15)\right]$$
$$X(f) = (j2)\left[\delta(f + 17.5/\pi - 15) + \delta(f + 17.5/\pi + 15) - \delta(f - 17.5/\pi - 15) - \delta(f - 17.5/\pi + 15)\right]$$

Nyquist rate is $f_{NYQ} = 2 \times (17.5/\pi + 15) = 41.141$

(d) $x(t) = \left[u(t) - u(t-3)\right]\sin(28\pi t)$ Time-limited, therefore not Bandlimited

(e) $x(t) = 4\cos(50\pi t) - 6\sin(68\pi t)$ Highest frequency is 34 Hz. $f_{NYQ} = 68$

4. A discrete-time system has a transfer function $H(z) = \dfrac{z^2 - 1}{z^2 + 0.95}$.

(a) At what numerical radian frequency or frequencies in the range $-\pi \le \Omega < \pi$ is the magnitude of this system's frequency response a minimum?

Minimum occurs at $z = \pm 1 \Rightarrow \Omega = -\pi, 0$

(b) At what numerical radian frequency or frequencies in the range $-\pi \le \Omega < \pi$ is the magnitude of this system's frequency response a maximum?

Maximum occurs at nearest approach to poles at $z = \pm j0.975 \Rightarrow \Omega = \pm\pi/2$

(c) What is the numerical magnitude <u>in dB</u> of the frequency response at $\Omega = 1.5$.

$$H(e^{j\Omega}) = \frac{e^{j2\Omega} - 1}{e^{j2\Omega} + 0.95} \Rightarrow H(e^{j2}) = \frac{e^{j3} - 1}{e^{j3} + 0.95} = 13.601e^{j1.2238} \Rightarrow \left|H(e^{j\Omega})\right|_{\text{dB}} = 22.67 \text{ dB}$$

5. The frequency response of a continuous-time LTI system is $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$. The magnitude Bode diagram for this system is displayed below. (The dashed lines are asymptotes.)

(a) List the numerical locations in radians/second of all finite poles.

Poles at 40 and 9000

(b) List the numerical locations in radians/second of all finite zeros.

Zero at 300

(c) A sinusoidal signal $x(t) = 3\sin(30000t)$ volts is applied to the system. What is the numerical amplitude of the sinusoidal response in volts?

Frequency response magnitude at $\omega = 30000$ is about -90 dB. So the output signal response amplitude $A$ can be found from

$$20\log_{10}(A/3) = -90 \Rightarrow A = 3\times 10^{-90/20} = 0.00009487 \text{ or } 9.487\times 10^{-5}$$