Solution of ECE 315 Test 13 F04

1. An LTI system has a transfer function,

$$\mathrm{H}(j\omega) = \frac{j3\omega - \omega^2}{1000 - 10\omega^2 + j250\omega} \ .$$

(a) Find all the corner frequencies (in radians per second) in a magnitude Bode plot of this transfer function.
Corner frequencies (radians/s): 3, 5, 20 (and 0)

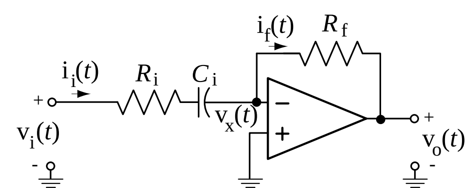
Corner frequencies are the magnitudes of the locations of the non-zero real poles and zeros and the radian resonant frequencies of any complex poles or zeros.

$$\mathrm{H}(j\omega) = \frac{j\omega(j\omega+3)}{10(j\omega)^2 + j250\omega + 1000} = \frac{1}{10} \times j\omega \times (j\omega + 3) \times \frac{1}{(j\omega+5)(j\omega+20)}$$

Zeros at $j\omega = 0$ and $j\omega = -3$. Poles at $j\omega = -5$ and $j\omega = -20$. (There is not actually a "corner" at zero but I accepted it anyway.)

(b) At very low and very high frequencies what is the slope of the magnitude Bode plot in dB/decade?
Slope = +20 dB/decade at very low frequencies and 0 dB/decade at very high frequencies
At very low freqencies the transfer function is approximately $\mathrm{H}(j\omega) = j3\omega/1000$. This is a frequency-independent gain times a single differentiator so the slope is +20 dB/decade. At very high frequencies the transfer function is approximately $\mathrm{H}(j\omega) = \frac{j\omega(j\omega)}{10(j\omega)^2} = \frac{1}{10}$ which is a constant so the slope is 0 dB/decade.

2. (a) Find the transfer function, $\mathrm{H}(j\omega) = \frac{\mathrm{V}_o(j\omega)}{\mathrm{V}_i(j\omega)}$, of this active filter with $R_i = 1000\ \Omega$, $C_i = 1\mu\mathrm{F}$ and $R_f = 5000\ \Omega$.



Using the gain formula for an inverting amplifier, $\mathrm{H}(j\omega) = -\frac{R_f}{R_i + \frac{1}{j\omega C_i}} = -\frac{j\omega R_f C_i}{j\omega R_i C_i + 1} = -\frac{j5\times10^{-3}\omega}{j10^{-3}\omega + 1}$.

(b) Find all the corner frequencies (in radians per second) in a magnitude Bode plot of this transfer function.
Corner frequencies (radians/s): 1000 (and 0)

There is one real pole at $j10^{-3}\omega + 1 = 0 \Rightarrow j\omega = -1000$ and one real zero at $j\omega = 0$. (Again, there is not actually a "corner" at zero but I accepted it anyway.)

(c) At very low and very high frequencies what is the slope of the magnitude Bode plot in dB/decade?
Slope = +20 dB/decade at very low frequencies and 0 dB/decade at very high frequencies

At very low freqencies the transfer function is approximately $\mathrm{H}(j\omega) = -j5\times10^{-3}\omega$. This is a single differentiator times a frequency-independent gain of -5×10^{-3} so the slope is +20 dB/decade. At very high frequencies the transfer function is approximately $\mathrm{H}(j\omega) = -5$ which is a constant so the slope is 0 dB/decade.

Solution of ECE 315 Test 13 F04

1. An LTI system has a transfer function,

$$\mathrm{H}(j\omega) = \frac{1200 - 10\omega^2 + j340\omega}{j6\omega - \omega^2} .$$

(a) Find all the corner frequencies (in radians per second) in a magnitude Bode plot of this transfer function.
Corner frequencies (radians/s): <u>6, 4, 30 (and 0)</u>

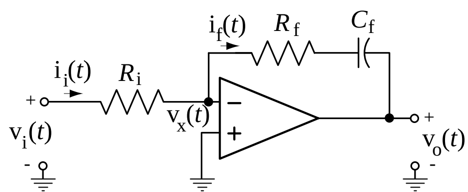
Corner frequencies are the magnitudes of the locations of the non-zero real poles and zeros and the radian resonant frequencies of any complex poles or zeros.

$$\mathrm{H}(j\omega) = \frac{10(j\omega)^2 + j340\omega + 1200}{j\omega(j\omega+6)} = 10 \times (j\omega+4)(j\omega+30) \times \frac{1}{j\omega} \times \frac{1}{j\omega+6}$$

Zeros at $j\omega = -4$ and $j\omega = -30$. Poles at $j\omega = 0$ and $j\omega = -6$. (There is not actually a "corner" at zero but I accepted it anyway.)

(b) At very low and very high frequencies what is the slope of the magnitude Bode plot in dB/decade?
Slope = <u>-20</u> dB/decade at very low frequencies and <u>0</u> dB/decade at very high frequencies
At very low freqencies the transfer function is approximately $\mathrm{H}(j\omega) = 200 / j\omega$. This is a frequency-independent gain times a single integrator so the slope is -20 dB/decade. At very high frequencies the transfer function is approximately $\mathrm{H}(j\omega) = 10$ which is a constant so the slope is 0 dB/decade.

2. (a) Find the transfer function, $\mathrm{H}(j\omega) = \frac{\mathrm{V}_o(j\omega)}{\mathrm{V}_i(j\omega)}$, of this active filter with $R_i = 2000\ \Omega$, $C_f = 1\mu\mathrm{F}$ and $R_f = 4000\ \Omega$.



Using the gain formula for an inverting amplifier, $\mathrm{H}(j\omega) = -\frac{R_f + \frac{1}{j\omega C_f}}{R_i} = -\frac{j\omega R_f C_f + 1}{j\omega R_i C_f} = -\frac{j4\times10^{-3}\omega + 1}{j2\times10^{-3}\omega}$.

(b) Find all the corner frequencies (in radians per second) in a magnitude Bode plot of this transfer function.
Corner frequencies (radians/s): <u>250 (and 0)</u>

There is one real zero at $j4\times10^{-3}\omega + 1 = 0 \Rightarrow j\omega = -250$ and one real pole at $j\omega = 0$. (Again, there is not actually a "corner" at zero but I accepted it anyway.)

(c) At very low and very high frequencies what is the slope of the magnitude Bode plot in dB/decade?
Slope = <u>-20</u> dB/decade at very low frequencies and <u>0</u> dB/decade at very high frequencies

At very low freqencies the transfer function is approximately $\mathrm{H}(j\omega) = -500 / j\omega$. This is a single integrator times a frequency-independent gain of -500 so the slope is -20 dB/decade. At very high frequencies the transfer function is approximately $\mathrm{H}(j\omega) = -2$ which is a constant so the slope is 0 dB/decade.