Solution of ECE 316 Test #2 Su03

1. (3 pts each) A unity-gain-feedback system with overall transfer function, $\mathrm{H}(s)$, has a forward path transfer function, $\mathrm{H}_1(s)$. For each of the following forward-path transfer functions, assuming the overall system is stable, determine whether the responses to a unit-step and unit-ramp excitation of the overall unity-gain feedback system have zero, finite or infinite steady-state error. Circle correct answers.

		Unit-step Excitation	Unit-Ramp Excitation
(a)	$\mathrm{H}_1(s)=\dfrac{10s}{s+10}$	Finite	Infinite

Type 0 System

(b)	$\mathrm{H}_1(s)=\dfrac{-7s}{(s+4)(s+12)}$	Finite	Infinite

Type 0 System

(c)	$\mathrm{H}_1(s)=\dfrac{(s+5)(s+8)}{s^2(s+1)(s+25)}$	Zero	Zero

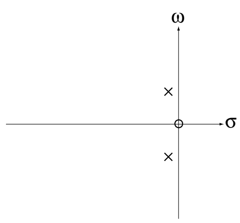
Type 2 System

(d)	$\mathrm{H}_1(s)=\dfrac{1}{s(s+11)(s+32)}$	Zero	Finite

Type 1 System

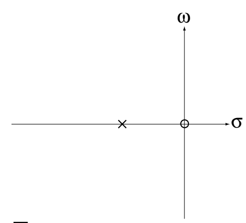
2. (3 pts each) Classify each of the systems with the following pole-zero diagrams as practical approximations to a lowpass (LP), highpass (HP), bandpass (BP) or bandstop (BS) filter.

(a) BP



Zero at zero
Two pole vectors and one zero vector
Therefore zero at infinity.

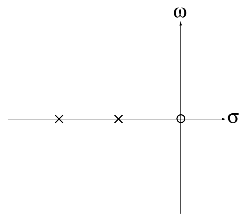
(b) HP



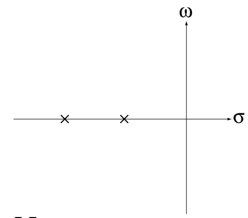
Zero at zero
One pole vector and one zero vector
Therefore non-zero at infinity

(c) BP

(d) LP

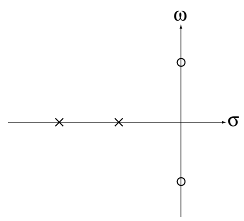


Zero at zero
Two pole vectors and one zero vector
Therefore zero at infinity.



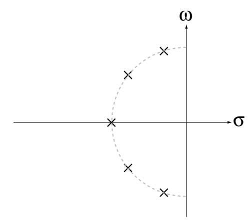
Non-zero at zero
Two pole vectors and no zero vectors
Therefore zero at infinity

(e) BS



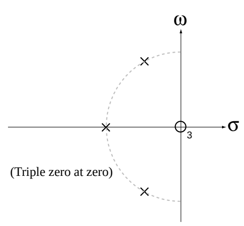
Two finite zeros at a finite frequency
Two pole vectors and two zero vectors
Therefore non-zero at infinity

(f) LP



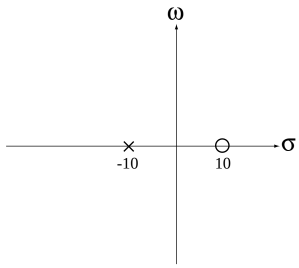
Non-zero at zero
5 pole vectors and no zero vectors
Therefore zero at infinity

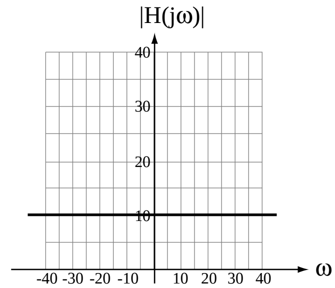
(g) HP

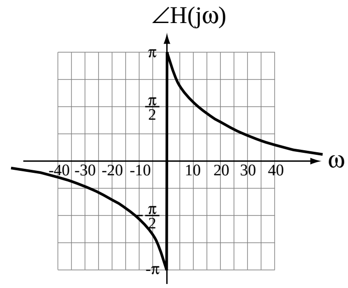


3 zeros at zero
3 pole vectors and 3 zero vectors
Therefore non-zero at infinity

3. (15 pts) Below is pole-zero diagram for a transfer function, $\mathrm{H}(s)$. Given that $\mathrm{H}(0) = -10$ sketch a graph of the magnitude frequency response, $|\mathrm{H}(j\omega)|$, and phase frequency response, $\angle\mathrm{H}(j\omega)$, as functions of ω in the spaces provided below. The sketch should be roughly the right shape and have the correct limiting behavior at low and high frequencies. (Plot a correct phase response in the range provided, $-\pi < \angle\mathrm{H}(j\omega) < \pi$.)







$$\mathrm{H}(s)=\frac{10(s-10)}{s+10}\Rightarrow\left|\mathrm{H}(j\omega)\right|=\left|\frac{10(j\omega-10)}{j\omega+10}\right|=10$$

At any frequency the zero vector and the pole vector are always the same length so the transfer function magnitude is constant.

$$\mathrm{H}(s)=\frac{10(s-10)}{s+10}\Rightarrow\angle\mathrm{H}(j\omega)=\angle 10+\angle(s-10)-\angle(s+10)$$

At zero frequency the phase is π radians ($\mathrm{H}(0)=-10$). As ω increases from zero, the zero vector angle rotates clockwise, moving in the negative direction, and the pole vector rotates counterclockwise, moving in the positive direction. Both these angular changes contribute to the phase going negative from π radians. As ω approaches infinity the two vectors approach the same angle and the net angle is zero.

4. (10 pts) The excitation signal, $\mathrm{x}(t)=20\cos(40\pi t)\mathrm{u}(t)$, is applied to a system whose transfer function is $\mathrm{H}(s)=\dfrac{5}{s+150}$. The response contains a transient term and a steady-state term. After the transient term has died away what is the amplitude, $A_{response}$, of the response and what is the phase difference between the excitation and response signals, $\Delta\theta=\theta_{excitation}-\theta_{response}$?

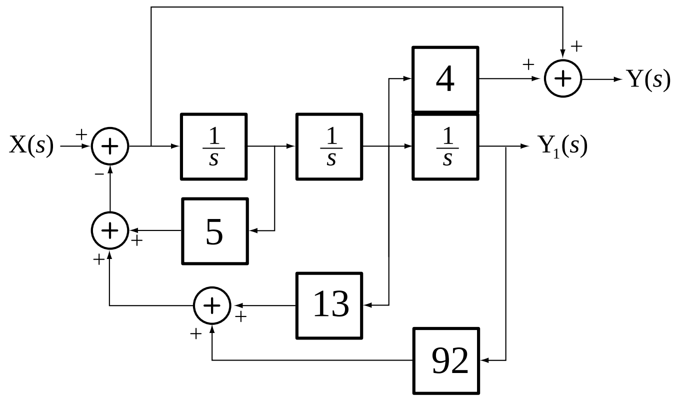
$$\mathrm{y}(t)=L^{-1}\left(\frac{\mathrm{N}_1(s)}{\mathrm{D}(s)}\right)+\left|\mathrm{H}(j\omega_0)\right|\cos\left(\omega_0 t+\angle\mathrm{H}(j\omega_0)\right)\mathrm{u}(t)$$

Amplitude is $20\times\left|\mathrm{H}(j40\pi)\right|=20\times\left|\dfrac{5}{j40\pi+150}\right|=\dfrac{100}{\sqrt{(40\pi)^2+150^2}}=0.511$

Phase difference, $\Delta\theta=\theta_{excitation}-\theta_{response}$, is $-\angle\mathrm{H}(j40\pi)=\angle(j40\pi+150)=0.697$

5. (10 pts) Draw a canonical system diagram of the system with the transfer function,

$$\mathrm{H}(s)=\frac{s\left(s^2+4\right)}{s^3+5s^2+13s+92}\ .$$



6. (10 pts) Draw a parallel system diagram of the system with the transfer function,

$$\mathrm{H}(s)=\frac{s^3}{(s+4)(s+10)(s+6)}\ .$$

$$\mathrm{H}(s)=\frac{s^3}{(s+4)(s+10)(s+6)}=\frac{s^3}{\left(s^2+14s+40\right)(s+6)}=\frac{s^3}{s^3+20s^2+124s+240}$$

$$\mathrm{H}(s)=1-\frac{20s^2+124s+240}{s^3+20s^2+124s+240}=1-\frac{20s^2+124s+240}{(s+4)(s+10)(s+6)}$$

$$\mathrm{H}(s)=1-\left(\frac{\frac{16}{3}}{s+4}+\frac{\frac{125}{3}}{s+10}-\frac{27}{s+6}\right)$$