Solution of ECE 316 Test 3 Su08

1. Find the numerical values of the constants.

(a) $$4e^{-5t}\cos(25\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} A\frac{s+a}{s^2+bs+c}$$

$$4e^{-5t}\cos(25\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 4\frac{s+5}{(s+5)^2+(25\pi)^2}$$

$$4e^{-5t}\cos(25\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 4\frac{s+5}{s^2+10s+25+(25\pi)^2} = 4\frac{s+5}{s^2+10s+6193.5}$$

(b) $$\frac{6}{(s+4)(s+a)} = \frac{2}{s+4}+\frac{b}{s+a}$$

$$\frac{6}{(s+4)(s+a)} = \frac{\frac{6}{-4+a}}{s+4}+\frac{\frac{6}{-a+4}}{s+a} = \frac{\frac{6}{a-4}}{s+4}+\frac{\frac{6}{4-a}}{s+a}$$

$$\frac{2}{s+4}+\frac{b}{s+a} = \frac{\frac{6}{a-4}}{s+4}+\frac{\frac{6}{4-a}}{s+a} \Rightarrow \frac{6}{a-4}=2 \text{ and } \frac{6}{4-a}=b$$

$$\frac{6}{a-4}=2 \Rightarrow a=7$$

$$\frac{6}{4-7}=b=-2$$

$$\frac{6}{(s+4)(s+7)} = \frac{2}{s+4}+\frac{-2}{s+7} \quad \text{Check.}$$

2. Find the numerical values of the constants.

(a) $$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 3\frac{3s+4}{s^2+9}$$

$$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{9s+12}{s^2+9} = 9\frac{s}{s^2+9}+4\frac{3}{s^2+9}$$

$$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 9\cos(3t)\mathrm{u}(t)+4\sin(3t)\mathrm{u}(t)$$

$$A=4 \;\;,\;\; B=9 \;\;,\;\; a=3$$

(b) $$Ae^{-at}\left[\sin(bt)+B\cos(bt)\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{35s+325}{s^2+18s+85}$$

$$\frac{35s+325}{s^2+18s+85} = 5\frac{7s+65}{(s+9)^2+4} = 5\left[\frac{2}{(s+9)^2+4}+7\frac{s+9}{(s+9)^2+4}\right]$$

$$5e^{-9t}\left[\sin(2t)+7\cos(2t)\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{35s+325}{s^2+18s+85}$$

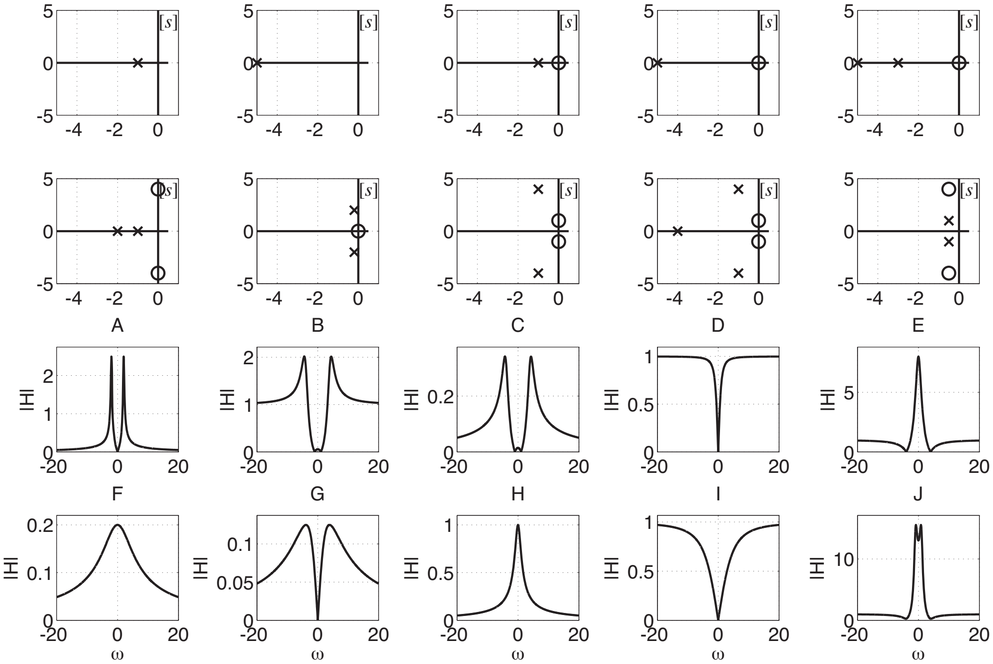
3. Match the pole-zero diagrams to the magnitude frequency responses. (Assume that the transfer functions are of the form

$$H(s) = A\frac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

and that $A = 1$.) In each case all finite poles and zeros are shown.

H F D I G

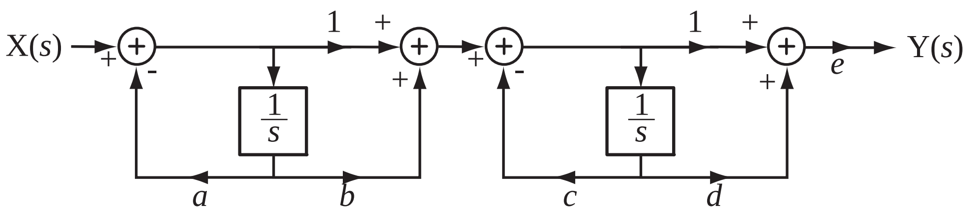
E A B C J



4. A system has a transfer function

$$H(s) = \frac{5s^2 + 40s}{s^2 + 11s + 30}$$

Find numerical values of the constants *a* through *e* below to realize this transfer function. (There is more than one correct way to choose the constants.)



$$H(s) = \frac{5s^2 + 40s}{s^2 + 11s + 30} = 5 \times \frac{s}{s+5} \times \frac{s+8}{s+6}$$

$$a = 5 \;,\; b = 0 \;,\; c = 6 \;,\; d = 8 \;,\; e = 5$$

or

$$a = 6 \;,\; b = 0 \;,\; c = 5 \;,\; d = 8 \;,\; e = 5$$

or

$$a = 5 \;,\; b = 8 \;,\; c = 6 \;,\; d = 0 \;,\; e = 5$$

or

$$a = 6 \;,\; b = 8 \;,\; c = 5 \;,\; d = 0 \;,\; e = 5$$

Solution of ECE 316 Test 3 Su08

1. Find the numerical values of the constants.

(a) $$8e^{-2t}\cos(10\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} A\frac{s+a}{s^2+bs+c}$$

$$8e^{-2t}\cos(10\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 8\frac{s+2}{(s+2)^2+(10\pi)^2}$$

$$8e^{-2t}\cos(10\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 8\frac{s+2}{s^2+4s+4+(10\pi)^2} = 8\frac{s+2}{s^2+4s+990.96}$$

(b) $$\frac{4}{(s+6)(s+a)} = \frac{10}{s+6} + \frac{b}{s+a}$$

$$\frac{4}{(s+6)(s+a)} = \frac{\frac{4}{-6+a}}{s+6} + \frac{\frac{4}{-a+6}}{s+a} = \frac{\frac{4}{a-6}}{s+4} + \frac{\frac{4}{6-a}}{s+a}$$

$$\frac{10}{s+4} + \frac{b}{s+a} = \frac{\frac{4}{a-6}}{s+4} + \frac{\frac{4}{6-a}}{s+a} \Rightarrow \frac{4}{a-6} = 10 \text{ and } \frac{4}{6-a} = b$$

$$\frac{4}{a-6} = 10 \Rightarrow a = 6.4$$

$$\frac{4}{6-6.4} = b = -10$$

$$\frac{6}{(s+4)(s+7)} = \frac{2}{s+4} + \frac{-2}{s+7}$$ Check.

2. Find the numerical values of the constants.

(a) $$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}5\frac{2s+12}{s^2+16}$$

$$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}\frac{10s+60}{s^2+16}=10\frac{s}{s^2+16}+15\frac{4}{s^2+16}$$

$$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}10\cos(4t)\mathrm{u}(t)+15\sin(4t)\mathrm{u}(t)$$

$$A=15\ ,\ B=10\ ,\ a=4$$

(b) $$Ae^{-at}\left[\sin(bt)+B\cos(bt)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}\frac{48s+258}{s^2+10s+34}$$

$$\frac{48s+258}{s^2+10s+34}=6\frac{8s+43}{(s+5)^2+9}=6\left[\frac{3}{(s+5)^2+9}+8\frac{s+5}{(s+5)^2+9}\right]$$

$$6e^{-5t}\left[\sin(3t)+8\cos(3t)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}\frac{48s+258}{s^2+10s+34}$$

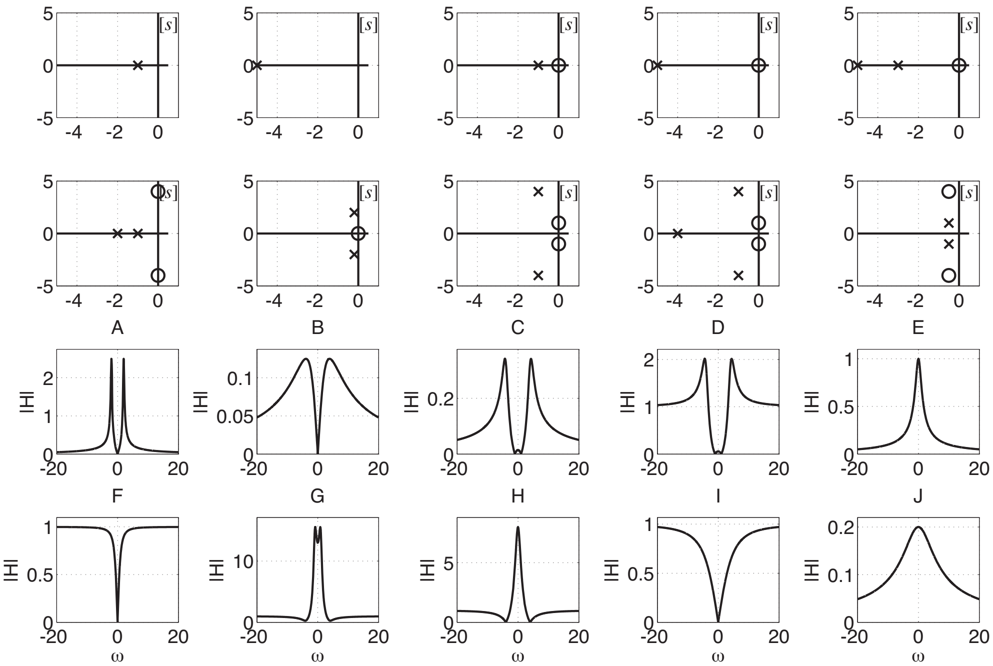
3. Match the pole-zero diagrams to the magnitude frequency responses. (Assume that the transfer functions are of the form

$$\mathrm{H}(s) = A\frac{(s - z_1)(s - z_2)\cdots(s - z_M)}{(s - p_1)(s - p_2)\cdots(s - p_N)}$$

and that $A = 1$.) In each case all finite poles and zeros are shown.

E J F I B

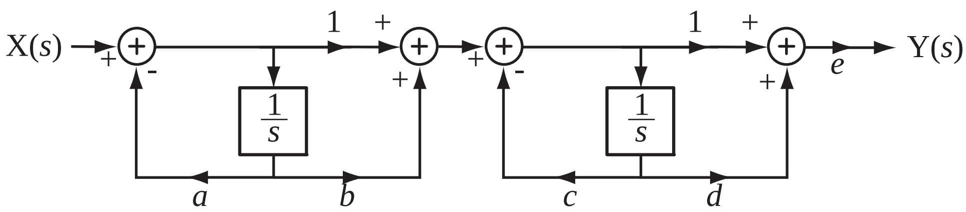
H A D C G



4. A system has a transfer function

$$H(s) = \frac{7s^2 + 28s}{s^2 + 9s + 14}$$

Find numerical values of the constants *a* through *e* below to realize this transfer function. (There is more than one correct way to choose the constants.)



$$H(s) = \frac{7s^2 + 28s}{s^2 + 9s + 14} = 7 \times \frac{s}{s+7} \times \frac{s+4}{s+2}$$

$$a = 7\ ,\ b = 0\ ,\ c = 2\ ,\ d = 4\ ,\ e = 7$$

or

$$a = 2\ ,\ b = 0\ ,\ c = 7\ ,\ d = 4\ ,\ e = 7$$

or

$$a = 7\ ,\ b = 4\ ,\ c = 2\ ,\ d = 0\ ,\ e = 7$$

or

$$a = 2\ ,\ b = 4\ ,\ c = 7\ ,\ d = 0\ ,\ e = 7$$

Solution of ECE 316 Test 3 Su08

1. Find the numerical values of the constants.

(a) $$12e^{-6t}\cos(40\pi t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} A\frac{s+a}{s^2+bs+c}$$

$$12e^{-6t}\cos(40\pi t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 12\frac{s+6}{(s+6)^2+(40\pi)^2}$$

$$12e^{-6t}\cos(40\pi t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 12\frac{s+6}{s^2+12s+36+(40\pi)^2} = 12\frac{s+6}{s^2+12s+15827.4}$$

(b) $$\frac{8}{(s+4)(s+a)} = \frac{2}{s+4} + \frac{b}{s+a}$$

$$\frac{8}{(s+4)(s+a)} = \frac{\frac{8}{-4+a}}{s+4} + \frac{\frac{8}{-a+4}}{s+a} = \frac{\frac{8}{a-4}}{s+4} + \frac{\frac{8}{4-a}}{s+a}$$

$$\frac{2}{s+4} + \frac{b}{s+a} = \frac{\frac{8}{a-4}}{s+4} + \frac{\frac{8}{4-a}}{s+a} \Rightarrow \frac{8}{a-4} = 2 \text{ and } \frac{8}{4-a} = b$$

$$\frac{8}{a-4} = 2 \Rightarrow a = 8$$

$$\frac{8}{4-8} = b = -2$$

$$\frac{8}{(s+4)(s+8)} = \frac{2}{s+4} + \frac{-2}{s+8} \quad \text{Check.}$$

2. Find the numerical values of the constants.

(a) $$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}4\frac{9s+3}{s^2+25}$$

$$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}\frac{36s+12}{s^2+25}=36\frac{s}{s^2+25}+2.4\frac{5}{s^2+25}$$

$$\left[A\sin(at)+B\cos(at)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}36\cos(5t)\mathrm{u}(t)+2.4\sin(5t)\mathrm{u}(t)$$

$$A=2.4\ ,\ B=36\ ,\ a=5$$

(b) $$Ae^{-at}\left[\sin(bt)+B\cos(bt)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}\frac{12s+126}{s^2+16s+89}$$

$$\frac{12s+126}{s^2+16s+89}=6\frac{2s+21}{(s+8)^2+25}=6\left[\frac{5}{(s+8)^2+25}+2\frac{s+8}{(s+8)^2+25}\right]$$

$$6e^{-8t}\left[\sin(5t)+2\cos(5t)\right]\mathrm{u}(t)\xleftrightarrow{\mathcal{L}}\frac{12s+126}{s^2+16s+89}$$

3. Match the pole-zero diagrams to the magnitude frequency responses. (Assume that the transfer functions are of the form

$$\mathrm{H}(s) = A\frac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

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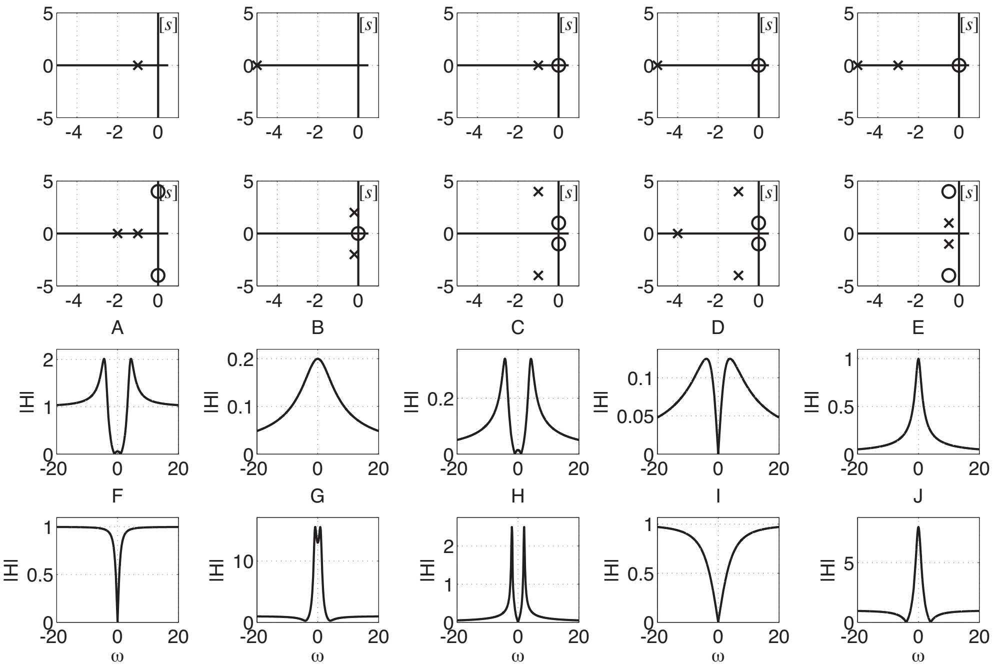
E	B	F	I	D
J	H	A	C	G



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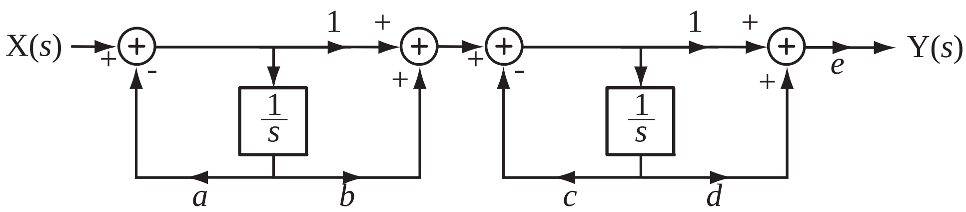

$$H(s)=\frac{2s^2+30s}{s^2+10s+9}=2\times\frac{s}{s+1}\times\frac{s+15}{s+9}$$

$$a=1\ ,\ b=0\ ,\ c=9\ ,\ d=15\ ,\ e=2$$

or

$$a=9\ ,\ b=0\ ,\ c=1\ ,\ d=15\ ,\ e=2$$

or

$$a=1\ ,\ b=15\ ,\ c=9\ ,\ d=0\ ,\ e=2$$

or

$$a=9\ ,\ b=15\ ,\ c=1\ ,\ d=0\ ,\ e=2$$