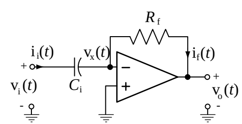
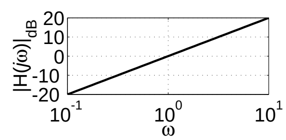
Solution to ECE Test #13 F03

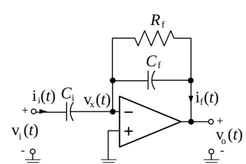
In the active filters below all resistors are 1 ohm and all capacitors are 1 farad. For each filter the transfer function is $H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)}$. Identify the transfer-function magnitude Bode diagram for each circuit by entering the correct letter.

1. 

$|H(j\omega)|$ is $\left|\frac{R_f}{\frac{1}{j\omega C_i}}\right|$. At very low frequencies that ratio approaches zero (negative infinity in dB) and at very high frequencies it approaches infinity (positive infinity in dB).

$$H(j\omega) = -\frac{R_f}{\frac{1}{j\omega C_i}} = -j\omega R_f C_i \Rightarrow \text{Differentiator}$$

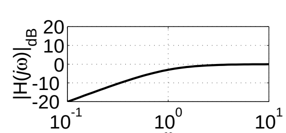


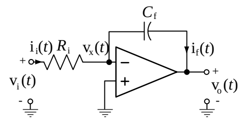
2. 

At very low frequencies $|H(j\omega)|$ is $\left|\frac{R_f}{\frac{1}{j\omega C_i}}\right|$ which approaches zero (negative infinity in dB).

At very high frequencies it is $\left|\frac{\frac{1}{j\omega C_f}}{\frac{1}{j\omega C_i}}\right|$ which is $\frac{C_i}{C_f} = 1$ or 0 dB.

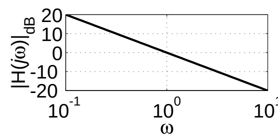
$$H(j\omega) = -\frac{\frac{R_f \frac{1}{j\omega C_f}}{R_f + \frac{1}{j\omega C_f}}}{\frac{1}{j\omega C_i}} = -\frac{j\omega R_f C_i}{j\omega R_f C_f + 1} \Rightarrow \text{Highpass Filter}$$

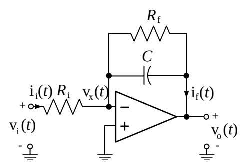


3. 

$\left|H(j\omega)\right|$ is $\left|\dfrac{\frac{1}{j\omega C_f}}{R_i}\right|$. At very low frequencies that ratio approaches infinity (positive infinity in dB) and at very high frequencies it approaches zero (negative infinity in dB).

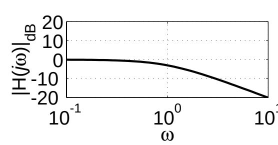
$$H(j\omega) = -\frac{\frac{1}{j\omega C_f}}{R_i} = -\frac{1}{j\omega R_i C_f} \Rightarrow \text{Integrator}$$

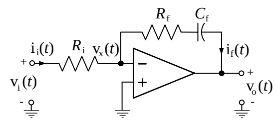


4. 

At very low frequencies $\left|H(j\omega)\right|$ approaches $\dfrac{R_f}{R_i} = 1$ or 0 dB. At very high frequencies it approaches $\left|\dfrac{\frac{1}{j\omega C}}{R_i}\right| = 0$, (negative infinity in dB).

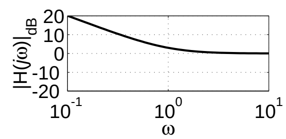
$$H(j\omega) = -\frac{\dfrac{R_f \frac{1}{j\omega C}}{R_f + \frac{1}{j\omega C}}}{R_i} = -\frac{R_f}{R_i}\frac{1}{j\omega R_f C + 1} \Rightarrow \text{Lowpass Filter}$$

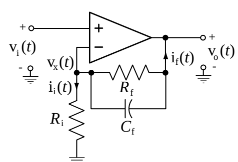


5. 

At very low frequencies $\left|H(j\omega)\right|$ approaches $\left|\dfrac{\frac{1}{j\omega C_f}}{R_i}\right|$ which approaches infinity (positive infinity in dB). At very high frequencies it approaches $\dfrac{R_f}{R_i} = 1$ or 0 dB.

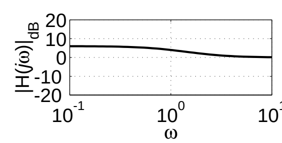
$$H(j\omega) = -\frac{R_f + \frac{1}{j\omega C_f}}{R_i} = -\frac{j\omega R_f C_f + 1}{j\omega R_i C_f}$$

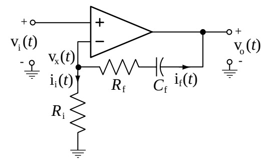


6. 

At very low frequencies $\left|H(j\omega)\right|$ approaches $\dfrac{R_f + R_i}{R_i} = 2$ or 6 dB. At very high frequencies it approaches $\left|\dfrac{\frac{1}{j\omega C_f} + R_i}{R_i}\right| = 1$ or 0 dB.

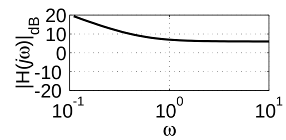
$$H(j\omega) = \frac{\dfrac{R_f \frac{1}{j\omega C_f}}{R_f + \frac{1}{j\omega C_f}} + R_i}{R_i} = \frac{j\omega R_i R_f C_f + \left(R_i + R_f\right)}{j\omega R_i R_f C_f + R_i}$$

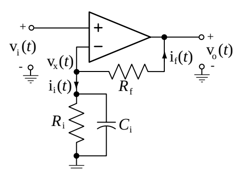


7. 

At very low frequencies $\left|\mathrm{H}(j\omega)\right|$ approaches $\left|\dfrac{\dfrac{1}{j\omega C_f}+R_i}{R_i}\right|$ which approaches infinity (positive infinity in dB). At very high frequencies it approaches $\dfrac{R_f+R_i}{R_i}=2$ or 6 dB.

$$\mathrm{H}(j\omega)=\frac{R_f+\dfrac{1}{j\omega C_f}+R_i}{R_i}=\frac{j\omega\left(R_f+R_i\right)C_f+1}{j\omega R_i C_f}$$



8. 

At very low frequencies $\left|\mathrm{H}(j\omega)\right|$ approaches $\dfrac{R_f+R_i}{R_i}=2$ or 6 dB. At very high frequencies it approaches $\left|\dfrac{R_f+\dfrac{1}{j\omega C_i}}{\dfrac{1}{j\omega C_i}}\right|$ which approaches infinity (positive infinity in dB).

$$\mathrm{H}(j\omega)=\frac{R_f+\dfrac{\dfrac{1}{j\omega C_i}R_i}{\dfrac{1}{j\omega C_i}+R_i}}{\dfrac{\dfrac{1}{j\omega C_i}R_i}{\dfrac{1}{j\omega C_i}+R_i}}=\frac{j\omega R_f R_i C_i+R_i+R_f}{R_i}$$