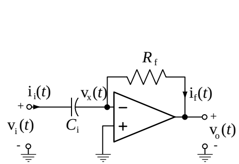
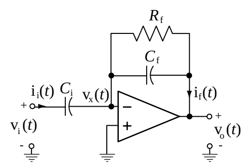
Solution to ECE 315 Final Examination Su04

1. In the active filters below all resistors are 1 ohm and all capacitors are 1 farad. For each filter the transfer function is $H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)}$. Identify the transfer-function magnitude Bode diagram for each circuit by entering the correct letter above the diagram.

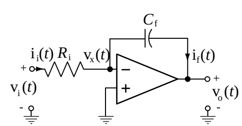
A



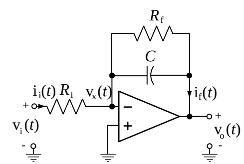
B



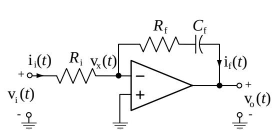
C



D



E



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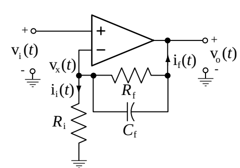

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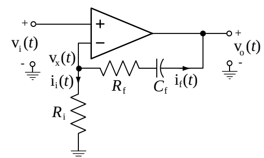

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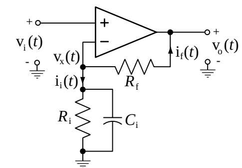

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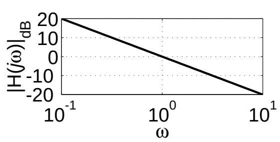

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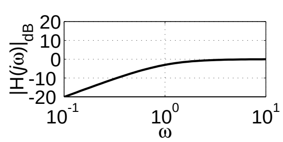

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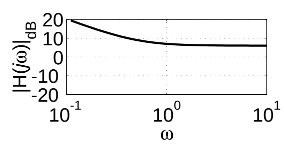

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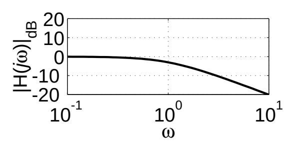
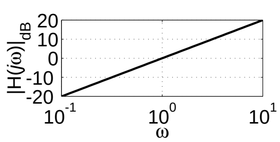


A



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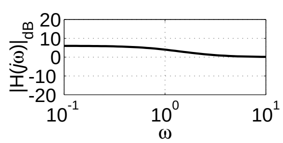

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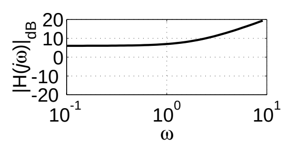

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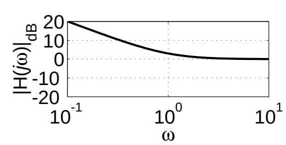
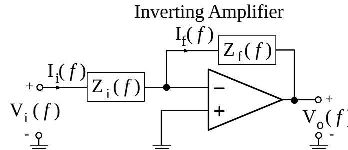
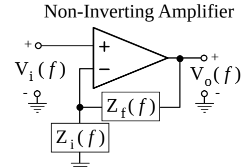


Inverting Amplifier



Non-Inverting Amplifier



$$\frac{V_o(f)}{V_i(f)} = \frac{Z_f(f) + Z_i(f)}{Z_i(f)}$$

2. What is the fundamental mathematical reason that it is physically impossible to build an ideal filter?

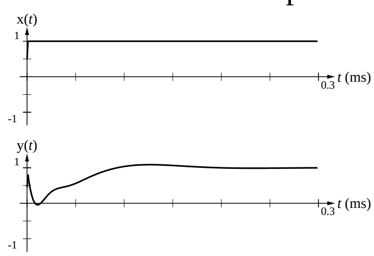
Every ideal filter has a non-causal impulse response.

3. There are only two things that can be done to a signal that change it without distorting it. What are they?

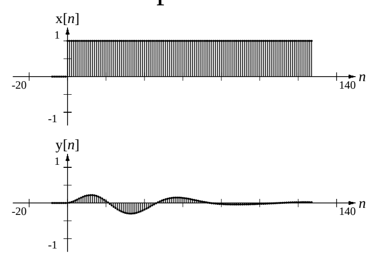
Shift it in time. Multiply it by a constant.

4. Below are pairs of excitations, x, and responses, y. For each pair, identify, on the line above the plot, the type of filtering that was done, lowpass, highpass, bandpass or bandstop.

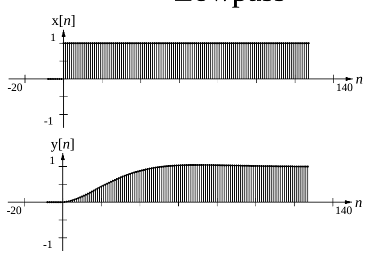
Bandstop



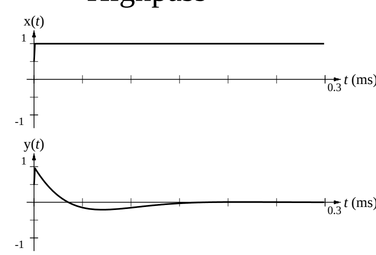
Bandpass



Lowpass



Highpass



5. Which of the three modulation techniques, DSBSC, DSBTC and SSBSC, require(s) synchronous demodulation to recover the original signal.

DSBSC SSBSC

6. One major problem in real instrumentation systems is electromagnetic interference caused by the 60 Hz power lines. A system with an impulse response of the form, $\mathrm{h}(t) = A(\mathrm{u}(t) - \mathrm{u}(t - t_0))$ can reject 60 Hz and all its harmonics. Find the numerical value of t_0 that makes this happen.

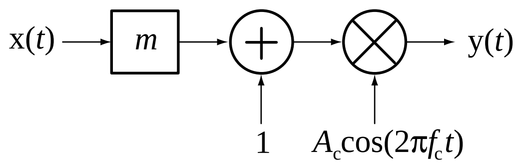
t_0 = 1/60 , $\mathrm{h}(t) = A(\mathrm{u}(t) - \mathrm{u}(t - t_0)) = A\,\mathrm{rect}\left(\dfrac{t - t_0/2}{t_0}\right)$

$\mathrm{H}(f) = At_0 \,\mathrm{sinc}(t_0 f) e^{-j\pi f t_0}$

This sinc function has nulls at integer multiples of $1/t_0$. If we want to reject 60 Hz and harmonics we must make $1/t_0 = 60$ which implies that $t_0 = 1/60$.

7. A sinusoid, $\mathrm{x}(t) = A_m \cos(2\pi f_m t)$, modulates a sinusoidal carrier, $A_c \cos(2\pi f_c t)$ in a double-sideband transmitted-carrier (DSBTC) system of the type illustrated below. If $A_m = 1$, $f_m = 10$, $A_c = 4$, $f_c = 1000$ and $m = 1$, find the numerical value of the total signal power in $\mathrm{y}(t)$ at the carrier frequency, P_c, and the numerical value of the total signal power in $\mathrm{y}(t)$ in its sidebands, P_s.

$P_c = 8$ $P_s = 4$



$$\mathrm{y}(t) = (m\,\mathrm{x}(t) + 1)A_c \cos(2\pi f_c t) = (mA_m \cos(2\pi f_m t) + 1)A_c \cos(2\pi f_c t)$$

$$\mathrm{y}(t) = 4(\cos(20\pi t) + 1)\cos(2000\pi t)$$

$$\mathrm{Y}(f) = 4\left\{\frac{1}{2}[\delta(f-10) + \delta(f+10)] + \delta(f)\right\} * \frac{1}{2}[\delta(f-1000) + \delta(f+1000)]$$

$$\mathrm{Y}(f) = 2\left\{\begin{array}{l}\frac{1}{2}[\delta(f-1010) + \delta(f-990)] + \delta(f-1000) \\ +\frac{1}{2}[\delta(f+990) + \delta(f+1010)] + \delta(f+1000)\end{array}\right\}$$

$$\mathrm{y}(t) = 2\cos(2020\pi t) + 2\cos(1980\pi t) + 4\cos(2000\pi t)$$

The signal power of the carrier is the signal power at 1000 Hz which is the signal power of $4\cos(2000\pi t)$. The signal power of any sinusoid is its the square of its amplitude, divided by 2. Therefore $P_c = 8$. The signal power in the sidebands is the signal power in $2\cos(2020\pi t) + 2\cos(1980\pi t)$. Since these are sinusoids at two different frequencies, the signal power of the sum is the sum of the signal powers which is $P_s = \frac{2^2}{2} + \frac{2^2}{2} = 4$.

8. A signal, $\mathrm{x}(t) = 4\,\mathrm{sinc}(10t)$ is the excitation of a single-sideband, suppressed-carrier (SSBSC) modulation system whose carrier is $10\cos(2000\pi t)$. The system generates the product of $\mathrm{x}(t)$ and the carrier to form a DSBSC signal, $\mathrm{y}_{DSBSC}(t)$. It then transmits the upper sideband and suppresses the lower sideband of $\mathrm{y}_{DSBSC}(t)$ with an ideal lowpass filter to form the transmitted signal, $\mathrm{y}(t)$. The transmitted signal, $\mathrm{y}(t)$, can be expressed in the form, $\mathrm{y}(t) = A\,\mathrm{sinc}(bt)\cos(ct)$. Find the numerical values of *A*, *b*, and *c*.

A = 20, *b* = 5, *c* = 2005π

$$\mathrm{y}_{DSBSC}(t) = 40\,\mathrm{sinc}(10t)\cos(2000\pi t)$$

$$\mathrm{Y}_{DSBSC}(f) = \frac{40}{10}\mathrm{rect}\left(\frac{f}{10}\right) * \frac{1}{2}\left[\delta(f-1000) + \delta(f+1000)\right]$$

$$\mathrm{Y}_{DSBSC}(f) = 2\left[\mathrm{rect}\left(\frac{f-1000}{10}\right) + \mathrm{rect}\left(\frac{f+1000}{10}\right)\right]$$

$$\mathrm{Y}(f) = 2\left[\mathrm{rect}\left(\frac{f-1002.5}{5}\right) + \mathrm{rect}\left(\frac{f+1002.5}{5}\right)\right]$$

$$\mathrm{y}(t) = 2\left[5\,\mathrm{sinc}(5t)e^{j2\pi(1002.5)t} + 5\,\mathrm{sinc}(5t)e^{-j2\pi(1002.5)t}\right]$$

$$\mathrm{y}(t) = 20\,\mathrm{sinc}(5t)\cos(2005\pi t)$$