

1. Find the regions of convergence of these unilateral Laplace transforms.

$$(a) \quad X(s) = \frac{s-4}{(s+2)(s+8)} \quad , \quad \sigma = \text{Re}(s) > -2$$

Poles at $s = -2$ and $s = -8$. Therefore the ROC is $\sigma > -2$.

$$(b) \quad X(s) = \frac{s(s+1)}{s^2 + s + 4} \quad , \quad \sigma = \text{Re}(s) > -0.5$$

Poles at $s = -0.5 \pm j1.9365$. Therefore the ROC is $\sigma > -0.5$

2. Find the unilateral Laplace transform of

$$x(t) = -2\delta(t-6) .$$

$$\delta(t) \xrightarrow{\mathcal{L}} 1 \quad , \quad \text{All } s \quad , \quad -2\delta(t) \xrightarrow{\mathcal{L}} -2 \quad , \quad \text{All } s \quad , \quad -2\delta(t-6) \xrightarrow{\mathcal{L}} -2e^{-6s} \quad , \quad \text{All } s$$

3. If the unilateral Laplace transform of $x(t) = e^{-5t} u(t)$ is $X(s)$, what are the numerical values of the magnitude and angle of $X(j2)$?

$$|X(j2)| = 0.1857 \quad , \quad \angle X(j2) = -0.3805 \text{ radians}$$

$$e^{-\alpha t} u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+\alpha} \quad , \quad \text{Re}(s) > -\alpha \quad , \quad e^{-5t} u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+5} \quad , \quad \text{Re}(s) > -5$$

$$X(s) = \frac{1}{s+5} \quad , \quad X(j2) = \frac{1}{j2+5} = \frac{5-j2}{25+4} = \frac{5}{29} - j\frac{2}{29} = 0.1858 \angle -0.3805$$