Solution to ECE Test #4 S07 #1

1. $\sin\left(\sqrt{8}t\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{\sqrt{8}}{s^2+8}$

$$\sin\left(\sqrt{8}t\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{\sqrt{8}}{3}\frac{3}{s^2+8}$$

$$\frac{3}{\sqrt{8}}\sin\left(\sqrt{8}t\right)\mathrm{u}(t) = 1.0607\sin(2.828t)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{3}{s^2+8}$$

In $A\sin(\omega_0 t)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{3}{s^2+8}$, $A = \underline{3/\sqrt{8} = 1.0607}$, $\omega_0 = \underline{\sqrt{8} = 2.828}$

2. $e^{-4t}\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{1}{s+4}$

$$\frac{d}{dt}\left(8e^{-4t}\mathrm{u}(t)\right) \overset{\mathcal{L}}{\longleftrightarrow} s\frac{8}{s+4} - \left[8e^{-4t}\mathrm{u}(t)\right]_{t=0^-} = \frac{8s}{s+4}$$

In $\dfrac{d}{dt}\left(8e^{-4t}\mathrm{u}(t)\right) \overset{\mathcal{L}}{\longleftrightarrow} K\dfrac{s+a}{s+b}$ $\qquad K = \underline{8}$, $a = \underline{0}$, $b = \underline{4}$

3.

$$\frac{2s+7}{(s+3)^2+4} = 2\frac{s+7/2}{(s+3)^2+4} = 2\left[\frac{s+3}{(s+3)^2+4} + \frac{1/2}{(s+3)^2+4}\right]$$

$$= 2\left[\frac{s+3}{(s+3)^2+4} + \frac{1/2}{2}\frac{2}{(s+3)^2+4}\right]$$

$$2\left[e^{-3t}\cos(2t) + (1/4)e^{-3t}\sin(2t)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} 2\left[\frac{s+3}{(s+3)^2+4} + \frac{1}{4}\frac{2}{(s+3)^2+4}\right]$$

$$2e^{-3t}\left[\cos(2t) + (1/4)\sin(2t)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} 2\left[\frac{s+3}{(s+3)^2+4} + \frac{1}{4}\frac{2}{(s+3)^2+4}\right]$$

In $Ke^{-at}\left[\cos(bt) + C\sin(ct)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{2s+7}{(s+3)^2+4}$

$K = \underline{2}$, $a = \underline{3}$, $b = \underline{2}$, $C = \underline{1/4=0.25}$, $c = \underline{2}$

Solution to ECE Test #4 S07 #2

1. $\sin\left(\sqrt{8}t\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{\sqrt{8}}{s^2+8}$

$$\sin\left(\sqrt{8}t\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{\sqrt{8}}{9}\frac{9}{s^2+8}$$

$$\frac{9}{\sqrt{8}}\sin\left(\sqrt{8}t\right)\mathrm{u}(t) = 3.182\sin(2.828t)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{9}{s^2+8}$$

In $A\sin(\omega_0 t)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{9}{s^2+8}$, $A = \underline{9/\sqrt{8}} = \underline{3.182}$, $\omega_0 = 2.828$

2. $e^{-7t}\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{1}{s+7}$

$$\frac{d}{dt}\left(3e^{-7t}\mathrm{u}(t)\right) \overset{\mathcal{L}}{\longleftrightarrow} s\frac{3}{s+7} - \left[3e^{-7t}\mathrm{u}(t)\right]_{t=0^-} = \frac{3s}{s+7}$$

In $\dfrac{d}{dt}\left(3e^{-7t}\mathrm{u}(t)\right) \overset{\mathcal{L}}{\longleftrightarrow} K\dfrac{s+a}{s+b}$ $\qquad K = \underline{3}$, $a = \underline{0}$, $b = \underline{7}$

3.

$$\frac{5s+9}{(s+2)^2+4} = 5\frac{s+9/5}{(s+2)^2+4} = 5\left[\frac{s+2}{(s+2)^2+4} - \frac{1/5}{(s+2)^2+4}\right]$$

$$= 5\left[\frac{s+2}{(s+2)^2+4} - \frac{1/5}{2}\frac{2}{(s+2)^2+4}\right]$$

$$5\left[e^{-2t}\cos(2t) - (1/10)e^{-2t}\sin(2t)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} 5\left[\frac{s+2}{(s+2)^2+4} - \frac{1}{10}\frac{2}{(s+2)^2+4}\right]$$

$$5e^{-2t}\left[\cos(2t) - (1/10)\sin(2t)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} 5\left[\frac{s+2}{(s+2)^2+4} - \frac{1}{10}\frac{2}{(s+2)^2+4}\right]$$

In $Ke^{-at}\left[\cos(bt) + C\sin(ct)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{5s+9}{(s+2)^2+4}$

$K = \underline{5}$, $a = \underline{2}$, $b = \underline{2}$, $C = \underline{-1/10}$, $c = \underline{2}$

Solution to ECE Test #4 S07 #3

1. $\sin\left(\sqrt{12}t\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{\sqrt{12}}{s^2+12}$

$$\sin\left(\sqrt{12}t\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{\sqrt{12}}{4}\frac{4}{s^2+12}$$

$$\frac{4}{\sqrt{12}}\sin\left(\sqrt{12}t\right)\mathrm{u}(t) = 1.155\sin(3.4641t)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{4}{s^2+12}$$

In $A\sin(\omega_0 t)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{4}{s^2+12}$ $\quad A = \underline{4/\sqrt{12}} = \underline{1.155}$, $\omega_0 = \underline{\sqrt{12}} = \underline{3.4641}$

2. $e^{-15t}\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{1}{s+15}$

$$\frac{d}{dt}\left(10e^{-15t}\mathrm{u}(t)\right) \overset{\mathcal{L}}{\longleftrightarrow} s\frac{10}{s+15} - \left[10e^{-15t}\mathrm{u}(t)\right]_{t=0^-} = \frac{10s}{s+15}$$

In $\dfrac{d}{dt}\left(10e^{-15t}\mathrm{u}(t)\right) \overset{\mathcal{L}}{\longleftrightarrow} K\dfrac{s+a}{s+b}$ $\quad K = \underline{10}$, $a = \underline{0}$, $b = \underline{15}$

3.

$$\frac{3s+5}{(s+4)^2+9} = 3\frac{s+5/3}{(s+4)^2+9} = 3\left[\frac{s+4}{(s+4)^2+9} - \frac{7/3}{(s+4)^2+9}\right]$$

$$= 3\left[\frac{s+4}{(s+4)^2+9} - \frac{7/3}{3}\frac{3}{(s+4)^2+9}\right]$$

$$3\left[e^{-4t}\cos(3t) - (7/9)e^{-4t}\sin(3t)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} 3\left[\frac{s+4}{(s+4)^2+9} - \frac{7}{9}\frac{3}{(s+4)^2+9}\right]$$

$$3e^{-4t}\left[\cos(3t) - (7/9)\sin(3t)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} 3\left[\frac{s+4}{(s+4)^2+9} - \frac{7}{9}\frac{3}{(s+4)^2+9}\right]$$

In $Ke^{-at}\left[\cos(bt) + C\sin(ct)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{3s+5}{(s+4)^2+9}$

$K = \underline{3}$, $a = \underline{4}$, $b = \underline{3}$, $C = \underline{-7/9 = -0.7778}$, $c = \underline{3}$