

Solution to ECE Test #6 S08 #1

1. Let $x(t) = 5e^{-t/2} \sin(\pi t / 6)u(t)$ and $x(t) \xrightarrow{\mathcal{L}} X(s)$. If $X(s) = \frac{A}{s^2 + bs + c}$ find the numerical values of A , b and c .

$$X(s) = 5 \frac{\pi / 6}{(s + 1/2)^2 + (\pi / 6)^2} = \frac{5\pi / 6}{s^2 + s + 1/4 + (\pi / 6)^2} = \frac{2.618}{s^2 + s + 0.524}$$

$A = 2.618$, $b = 1$, $c = 0.524$

2. Let $x(t) = 7\delta(t) - 4\delta(t - 5)$ and $x(t) \xrightarrow{\mathcal{L}} X(s)$. If $X(s) = A + Be^{cs} \sinh(ds)$ find the numerical values of A , B , c and d . ($\sinh(x) = \frac{e^x - e^{-x}}{2}$).

$$X(s) = 7 - 4e^{-5s} = 3 + 4(1 - e^{-5s}) = 3 + 4e^{-5s/2}(e^{5s/2} - e^{-5s/2}) = 3 + 8e^{-5s/2} \sinh(5s/2)$$

$A = 3$, $B = 8$, $c = -5/2$, $d = 5/2$

3. Let $x(t) = 3\cos(12t)u(t) * u(t)$. $x(t)$ can also be written in the form $x(t) = A \sin(bt)u(t)$. Find the numerical values of A and b .

$$X(s) = \frac{3s}{s^2 + 144} \times \frac{1}{s} = \frac{3}{s^2 + 144}$$

$$(1/4) \sin(12t) \xrightarrow{\mathcal{L}} \frac{3}{s^2 + 144} = \frac{1}{4} \frac{12}{s^2 + 144}$$

$A = 1/4$, $b = 12$

Solution to ECE Test #6 S08 #1

1. Let $x(t) = -3e^{-t/8} \sin(\pi t / 15)u(t)$ and $x(t) \xrightarrow{\mathcal{L}} X(s)$. If $X(s) = \frac{A}{s^2 + bs + c}$ find the numerical values of A , b and c .

$$X(s) = -3 \frac{\pi / 15}{(s + 1/8)^2 + (\pi / 15)^2} = \frac{-\pi / 5}{s^2 + s/4 + 1/64 + (\pi / 15)^2} = \frac{-0.628}{s^2 + s/4 + 0.0595}$$

$A = -0.628$, $b = 0.25$, $c = 0.0595$

2. Let $x(t) = 11\delta(t) - 7\delta(t - 3)$ and $x(t) \xrightarrow{\mathcal{L}} X(s)$. If $X(s) = A + Be^{cs} \sinh(ds)$ find the numerical values of A , B , c and d . ($\sinh(x) = \frac{e^x - e^{-x}}{2}$).

$$X(s) = 11 - 7e^{-3s} = 4 + 7(1 - e^{-3s}) = 4 + 7e^{-3s/2} (e^{3s/2} - e^{-3s/2}) = 4 + 14e^{-3s/2} \sinh(3s/2)$$

$A = 4$, $B = 14$, $c = -3/2$, $d = 3/2$

3. Let $x(t) = 8\cos(5t)u(t) * u(t)$. $x(t)$ can also be written in the form $x(t) = A \sin(bt)u(t)$. Find the numerical values of A and b .

$$X(s) = \frac{8s}{s^2 + 25} \times \frac{1}{s} = \frac{8}{s^2 + 25}$$

$$(8/5)\sin(5t) \xrightarrow{\mathcal{L}} \frac{8}{s^2 + 25} = \frac{8}{5} \frac{5}{s^2 + 25}$$

$A = 1.6$, $b = 5$