Solution to ECE Test 6 S09

Find the numerical values of the constants in the Laplace transform relationships below.

1. If $\frac{s^2+8}{4s^2+5s+7} = K_0 + \frac{K_1 s + K_2}{4s^2+5s+7}$ find the numerical values of K_0 , K_1 and K_2 .

$$\begin{array}{rl} & \qquad\qquad\qquad 1/4 \\ 4s^2+5s+7\big) & \overline{s^2 \qquad\qquad\quad +8} \\ & \underline{s^2+(5/4)s+7/4} \\ & \quad -(5/4)s+25/4 \end{array} \Rightarrow \frac{s^2+8}{4s^2+5s+7} = 1/4 + \frac{-(5/4)s+25/4}{4s^2+5s+7}$$

2. If $\frac{s-4}{(s+4)^2+25} = \frac{s+4}{(s+4)^2+25} + K_1 \frac{5}{(s+4)^2+25}$, find the numerical value of K_1 .

$$\frac{s-4}{(s+4)^2+25} = \frac{s+4}{(s+4)^2+25} - \frac{8}{(s+4)^2+25} = \frac{s+4}{(s+4)^2+25} + (-8/5)\frac{5}{(s+4)^2+25}$$

3. If $\frac{s+7}{s^2(s+3)} = \frac{K_{12}}{s^2} + \frac{K_{11}}{s} + \frac{K_2}{s+3}$, find the numerical values of K_{12} , K_{11} and K_2 .

$$K_{12} = \left[\frac{s+7}{s+3}\right]_{s=0} = \frac{7}{3} \;\;,\;\; K_{11} = \left[\frac{d}{ds}\left(\frac{s+7}{s+3}\right)\right]_{s=0} = \left[\frac{s+3-s-7}{(s+3)^2}\right]_{s=0} = -\frac{4}{9} \;\;,\;\; K_2 = \left[\frac{s+7}{s^2}\right]_{s=-3} = \frac{4}{9}$$

$$\frac{7/3}{s^2} - \frac{4/9}{s} + \frac{4/9}{s+3} = \frac{(7/3)(s+3) - (4/9)s(s+3) + (4/9)s^2}{s^2(s+3)} = \frac{s+7}{s^2(s+3)} \quad \text{Check.}$$

4. If $\frac{3s+12}{2s^2+16s+104} \xleftrightarrow{\mathscr{L}} Ae^{\alpha t}\cos(\beta t)\mathrm{u}(t)$, find the numerical values of A, α and β .

$$\frac{3}{2} \times \frac{s+4}{s^2+8s+52} \xleftrightarrow{\mathscr{L}} Ae^{\alpha t}\cos(\beta t)\mathrm{u}(t)$$

$$\frac{3}{2} \times \frac{s+4}{(s+4)^2+36} \xleftrightarrow{\mathscr{L}} (3/2)e^{-4t}\cos(6t)\mathrm{u}(t)$$

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Find the numerical values of the constants in the Laplace transform relationships below.

1. If $\dfrac{s^2+5}{3s^2+5s+7}=K_0+\dfrac{K_1 s+K_2}{3s^2+5s+7}$ find the numerical values of K_0 , K_1 and K_2.

$$\begin{array}{r|l} & \quad\quad\quad\quad\quad 1/3 \\ \hline 3s^2+5s+7 \,\big) & s^2 \quad\quad\quad +5 \\ & \underline{s^2+(5/3)s+7/3} \\ & -(5/3)s+8/3 \end{array} \Rightarrow \frac{s^2+5}{3s^2+5s+7}=1/3+\frac{-(5/3)s+8/3}{3s^2+5s+7}$$

2. If $\dfrac{s-2}{(s+4)^2+25}=\dfrac{s+4}{(s+4)^2+25}+K_1\dfrac{5}{(s+4)^2+25}$, find the numerical value of K_1.

$$\frac{s-2}{(s+4)^2+25}=\frac{s+4}{(s+4)^2+25}-\frac{6}{(s+4)^2+25}=\frac{s+4}{(s+4)^2+25}+(-6/5)\frac{5}{(s+4)^2+25}$$

3. If $\dfrac{s+4}{s^2(s+3)}=\dfrac{K_{12}}{s^2}+\dfrac{K_{11}}{s}+\dfrac{K_2}{s+3}$, find the numerical values of K_{12} , K_{11} and K_2.

$$K_{12}=\left[\frac{s+4}{s+3}\right]_{s=0}=\frac{4}{3}\ ,\ K_{11}=\left[\frac{d}{ds}\left(\frac{s+4}{s+3}\right)\right]_{s=0}=\left[\frac{s+3-s-4}{(s+3)^2}\right]_{s=0}=-\frac{1}{9}\ ,\ K_2=\left[\frac{s+4}{s^2}\right]_{s=-3}=\frac{1}{9}$$

$$\frac{4/3}{s^2}-\frac{1/9}{s}+\frac{1/9}{s+3}=\frac{(4/3)(s+3)-(1/9)s(s+3)+(1/9)s^2}{s^2(s+3)}=\frac{s+4}{s^2(s+3)}\quad\text{Check.}$$

4. If $\dfrac{4s+12}{2s^2+12s+50}\xleftrightarrow{\mathscr{L}}Ae^{\alpha t}\cos(\beta t)\mathrm{u}(t)$, find the numerical values of A, α and β.

$$\frac{4}{2}\times\frac{s+3}{s^2+6s+25}\xleftrightarrow{\mathscr{L}}Ae^{\alpha t}\cos(\beta t)\mathrm{u}(t)$$

$$2\times\frac{s+3}{(s+3)^2+16}\xleftrightarrow{\mathscr{L}}2e^{-3t}\cos(4t)\mathrm{u}(t)$$

Solution to ECE Test 6 S09

Find the numerical values of the constants in the Laplace transform relationships below.

1. If $\dfrac{s^2+7}{2s^2+5s+7} = K_0 + \dfrac{K_1 s + K_2}{2s^2+5s+7}$ find the numerical values of K_0 , K_1 and K_2 .

$$\begin{array}{r} 1/2 \\ 2s^2+5s+7\overline{\big)\,s^2 \qquad\quad +7} \\ \underline{s^2+(5/2)s+7/2} \\ -(5/2)s+7/2 \end{array} \Rightarrow \frac{s^2+7}{2s^2+5s+7} = 1/2 + \frac{-(5/2)s+7/2}{2s^2+5s+7}$$

2. If $\dfrac{s-6}{(s+4)^2+25} = \dfrac{s+4}{(s+4)^2+25} + K_1 \dfrac{5}{(s+4)^2+25}$, find the numerical value of K_1 .

$$\frac{s-6}{(s+4)^2+25} = \frac{s+4}{(s+4)^2+25} - \frac{10}{(s+4)^2+25} = \frac{s+4}{(s+4)^2+25} + (-2)\frac{5}{(s+4)^2+25}$$

3. If $\dfrac{s+11}{s^2(s+3)} = \dfrac{K_{12}}{s^2} + \dfrac{K_{11}}{s} + \dfrac{K_2}{s+3}$, find the numerical values of K_{12} , K_{11} and K_2 .

$$K_{12} = \left[\frac{s+11}{s+3}\right]_{s=0} = \frac{11}{3} \ , \ K_{11} = \left[\frac{d}{ds}\left(\frac{s+11}{s+3}\right)\right]_{s=0} = \left[\frac{s+3-s-11}{(s+3)^2}\right]_{s=0} = -\frac{8}{9} \ , \ K_2 = \left[\frac{s+11}{s^2}\right]_{s=-3} = \frac{8}{9}$$

$$\frac{11/3}{s^2} - \frac{8/9}{s} + \frac{8/9}{s+3} = \frac{(11/3)(s+3) - (8/9)s(s+3) + (8/9)s^2}{s^2(s+3)} = \frac{s+11}{s^2(s+3)} \quad \text{Check.}$$

4. If $\dfrac{6s+30}{2s^2+20s+100} \xleftrightarrow{\mathscr{L}} Ae^{\alpha t}\cos(\beta t)\mathrm{u}(t)$, find the numerical values of A, α and β .

$$\frac{6}{2} \times \frac{s+5}{s^2+10s+50} \xleftrightarrow{\mathscr{L}} Ae^{\alpha t}\cos(\beta t)\mathrm{u}(t)$$

$$3 \times \frac{s+5}{(s+5)^2+25} \xleftrightarrow{\mathscr{L}} 3e^{-5t}\cos(5t)\mathrm{u}(t)$$