Solution to ECE Test #2 S05

1. Given the Laplace-transform pair $\mathrm{g}(t) \xleftrightarrow{\mathcal{L}} \dfrac{3s(s+5)}{(s-2)(s^2+2s+8)}$ and the fact that $\mathrm{g}(t)$ is continuous at $t=0$, complete the following Laplace-transform pairs.

(a) $$\frac{d}{dt}\mathrm{g}(t) \xleftrightarrow{\mathcal{L}} \frac{3s^2(s+5)}{(s-2)(s^2+2s+8)} - \underbrace{\mathrm{g}(0^-)}_{=\mathrm{g}(0^+)}$$

$$\mathrm{g}(0^+) = \lim_{s\to\infty} \frac{3s^2(s+5)}{(s-2)(s^2+2s+8)} = 3$$

$$\frac{d}{dt}\mathrm{g}(t) \xleftrightarrow{\mathcal{L}} \frac{3s^2(s+5)}{(s-2)(s^2+2s+8)} - 3$$

(b) $$\mathrm{g}(t-3) \xleftrightarrow{\mathcal{L}} \frac{3s(s+5)e^{-3s}}{(s-2)(s^2+2s+8)}$$

2. Does the final-value theorem apply to $\dfrac{8(s-2)}{s^2+2s+8}$? Yes

Explain your answer. There are two poles, both in the open left half-plane.

3. Find the Laplace transform $\mathrm{X}(s)$ of $\mathrm{x}(t) = \cos(20\pi t)\mathrm{u}(t) * \mathrm{u}(t)$. Then find the inverse transform of $\mathrm{X}(s)$ and express it in the form $\mathrm{y}(t)\mathrm{u}(t)$ without using the convolution operator, $*$. What is the function, $\mathrm{y}(t)$?

$$\cos(20\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{s}{s^2+(20\pi)^2}$$

$$\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}$$

$$\cos(20\pi t)\mathrm{u}(t) * \mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}\frac{s}{s^2+(20\pi)^2} = \frac{1}{s^2+(20\pi)^2} = \frac{1}{20\pi}\frac{20\pi}{s^2+(20\pi)^2}$$

$$\frac{1}{20\pi}\sin(20\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{20\pi}\frac{20\pi}{s^2+(20\pi)^2}$$

$$\mathrm{y}(t) = \frac{1}{20\pi}\sin(20\pi t)$$

Solution to ECE Test #2 S05 #1

1. Given the Laplace-transform pair $\mathrm{g}(t) \xleftrightarrow{\mathcal{L}} \dfrac{7s(s+2)}{(s+1)(s+4)(s+8)}$ and the fact that $\mathrm{g}(t)$ is continuous at $t=0$, complete the following Laplace-transform pairs.

(a) $$\frac{d}{dt}\mathrm{g}(t) \xleftrightarrow{\mathcal{L}} \frac{7s^2(s+2)}{(s+1)(s+4)(s+8)} - \underbrace{\mathrm{g}(0^-)}_{=\mathrm{g}(0^+)}$$

$$\mathrm{g}(0^+) = \lim_{s\to\infty} \frac{7s^2(s+2)}{(s+1)(s+4)(s+8)} = 7$$

$$\frac{d}{dt}\mathrm{g}(t) \xleftrightarrow{\mathcal{L}} \frac{7s(s+2)}{(s+1)(s+4)(s+8)} - 7$$

(b) $$\mathrm{g}(t-5) \xleftrightarrow{\mathcal{L}} \frac{7s(s+5)e^{-5s}}{(s-2)(s^2+2s+8)}$$

2. Does the final-value theorem apply to $\dfrac{8(s+2)}{s^2-2s+8}$? No

Explain your answer. There are two poles, both in the right half-plane.

3. Find the Laplace transform $\mathrm{X}(s)$ of $\mathrm{x}(t) = \cos(50\pi t)\mathrm{u}(t) * \mathrm{u}(t)$. Then find the inverse transform of $\mathrm{X}(s)$ and express it in the form $\mathrm{y}(t)\mathrm{u}(t)$ without using the convolution operator, $*$. What is the function, $\mathrm{y}(t)$?

$$\cos(50\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{s}{s^2+(50\pi)^2}$$

$$\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}$$

$$\cos(50\pi t)\mathrm{u}(t) * \mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}\frac{s}{s^2+(50\pi)^2} = \frac{1}{s^2+(50\pi)^2} = \frac{1}{50\pi}\frac{50\pi}{s^2+(50\pi)^2}$$

$$\frac{1}{50\pi}\sin(50\pi t)\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{50\pi}\frac{50\pi}{s^2+(50\pi)^2}$$

$$\mathrm{y}(t) = \frac{1}{50\pi}\sin(50\pi t)$$