Solution to ECE Test #2 Su05

1. For a unilateral Laplace transform, the part of the [s] plane to the right of the real part of the pole with the most positive real part is called region of convergence.

2. Under what conditions can a Fourier transform be found by substituting $j\omega$ for s in a Laplace transform?

If the region of convergence of the Laplace transform contains the ω axis.

3. In the frequency shifting property $\frac{1}{a}\mathrm{g}\left(\frac{t}{a}\right) \xleftrightarrow{\mathcal{L}} \mathrm{G}(as)$ of the unilateral Laplace transform, why must the constant a be positive?

If the constant a were negative the function $\frac{1}{a}\mathrm{g}\left(\frac{t}{a}\right)$ would be non-causal and the unique relationship between a function and its Laplace transform would be lost.

4. Find the numerical values of the constants.

(a) $$8e^{-t/5}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

a_2 = 0 a_1 = 1 a_0 = 0.2

b_2 = 0 b_1 = 0 b_0 = 8

$$8e^{-t/5}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{8}{s+0.2}$$

(b) $$-4\cos(20\pi t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

a_2 = 1 a_1 = 0 a_0 = 3947.8

b_2 = 0 b_1 = -4 b_0 = 0

$$-4\cos(20\pi t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{-4s}{s^2 + (20\pi)^2} = \frac{-4s}{s^2 + 3947.8}$$

(c) $$7e^{4t}\sin(300t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

a_2 = 1 a_1 = -8 a_0 = 90016

b_2 = 0 b_1 = 0 b_0 = 2100

$$7e^{4t}\sin(300t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 7 \times \frac{300}{(s-4)^2 + 9\times 10^4} = \frac{2100}{s^2 - 8s + 90016}$$

(d) $10\left(e^{-2t}+e^{-t/3}\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$

a_2 = 1 a_1 = 7/3 a_0 = 2/3

b_2 = 0 b_1 = 20 b_0 = 70/3

$$10\left(e^{-2t}+e^{-t/3}\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} 10\left(\frac{1}{s+2}+\frac{1}{s+1/3}\right) = 10\frac{2s+7/3}{s^2+7s/3+2/3} = \frac{20s+70/3}{s^2+7s/3+2/3}$$

(e) $\dfrac{s^2+3}{3s^2+s+9} = K_0 + \dfrac{K_1}{s-p_1} + \dfrac{K_2}{s-p_2}$

K_0 = 1/3 K_1 = -0.0556-j0.0054 p_1 = - 0.1667+j1.724

K_2 = -0.0556+j0.0054 p_2 = - 0.1667-j1.724

$$\frac{s^2+3}{3s^2+s+9} = \frac{1}{3} - \frac{s/3}{3s^2+s+9} = \frac{1}{3} - \frac{s/9}{s^2+s/3+3} = \frac{1}{3} - \left(\frac{0.0556+j0.0054}{s+0.1667-j1.724} + \frac{0.0556-j0.0054}{s+0.1667+j1.724}\right)$$

(f) $\left(K_1 t e^{at} + K_2 e^{bt} + K_3 e^{ct}\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{2(s-3)}{s^2(s+5)}$

K_1 = -1.2 K_2 = 0.64 K_3 = -0.64

a = 0 b = 0 c = -5

$$\frac{2(s-3)}{s^2(s+5)} = \frac{-6/5}{s^2} + \frac{16/25}{s} + \frac{-16/25}{s+5}$$

$$\left(-1.2t + 0.64 - 0.64e^{-5t}\right)\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{-6/5}{s^2} + \frac{16/25}{s} + \frac{-16/25}{s+5}$$

5. Given that $g(t) * u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{8(s+2)}{s(s+4)}$ find the function $g(t)$.

Using $u(t) \overset{\mathcal{L}}{\longleftrightarrow} 1/s$ and multiplication-convolution duality

$$g(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{8(s+2)}{s+4} = 8\left(1 - \frac{2}{s+4}\right)$$

$$g(t) = 8\delta(t) - 16e^{-4t}\,u(t) \overset{\mathcal{L}}{\longleftrightarrow} 8\left(1 - \frac{2}{s+4}\right)$$

6. Given that $g(t) * u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{5}{s+12}$ find the function $g(t)$.

Using $u(t) \overset{\mathcal{L}}{\longleftrightarrow} 1/s$ and multiplication-convolution duality

$$g(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{5s}{s+12} = 5 - \frac{60}{s+12}$$

$$g(t) = 5\delta(t) - 60e^{-12t}\,u(t) \overset{\mathcal{L}}{\longleftrightarrow} 5 - \frac{60}{s+12}$$