Solution of ECE 316 Test 4 S06

1. A causal signal $\mathrm{x}(t)$ has a Laplace transform $\mathrm{X}(s) = \frac{s}{s^4 - 16}$. If $\mathrm{y}(t) = 5\,\mathrm{x}(3t)$, $\mathrm{Y}(s)$ can be expressed in the form $\mathrm{Y}(s) = \frac{As}{s^4 - a^4}$. What are the numerical values of A and a? $A = 45$, $a = 6$

$$\mathrm{Y}(s) = 5 \times \frac{1}{3} \frac{s/3}{(s/3)^4 - 16} = \frac{5}{9} \frac{s}{s^4/81 - 16} = \frac{405}{9} \frac{s}{s^4 - 1296} = \frac{45s}{s^4 - 6^4}$$

2. What is the region of convergence of the Laplace transform of $\mathrm{x}(t) = \delta(t) - 2e^{-2t}\,\mathrm{u}(t)$?

$$\mathrm{X}(s) = 1 - \frac{2}{s+2} = \frac{s+2-2}{s+2} = \frac{s}{s+2}$$

Zero at $s = 0$ and a pole at $s = -2$. Region to the right of the pole is the region of convergence. That is $\mathrm{Re}(s) = \sigma > -2$.

3. Using the differentiation properties of the Laplace transform write the Laplace transform of the differential equation

$$\mathrm{x}''(t) - 2\,\mathrm{x}'(t) + 4\,\mathrm{x}(t) = \mathrm{u}(t)$$

$$s^2\,\mathrm{X}(s) - s\,\mathrm{x}(0^-) - \left(\frac{d}{dt}(\mathrm{x}(t))\right)_{t=0^-} - 2\left[s\,\mathrm{X}(s) - \mathrm{x}(0^-)\right] + 4\,\mathrm{X}(s) = \frac{1}{s}\ .$$

Solution of ECE 316 Test 4 S06

1. A causal signal $x(t)$ has a Laplace transform $X(s)=\frac{s}{s^4-1}$. If $y(t)=3x(2t)$, $Y(s)$ can be expressed in the form $Y(s)=\frac{As}{s^4-a^4}$. What are the numerical values of A and a? $A = 12$, $a = 2$

$$Y(s)=3\times\frac{1}{2}\frac{s/2}{(s/2)^4-1}=\frac{3}{4}\frac{s}{s^4/16-1}=12\frac{s}{s^4-16}=\frac{12s}{s^4-2^4}$$

2. What is the region of convergence of the Laplace transform of $x(t)=\delta(t)+2e^{2t}u(t)$?

$$X(s)=1+\frac{2}{s-2}=\frac{s-2+2}{s-2}=\frac{s}{s-2}$$

Zero at $s=0$ and a pole at $s=2$. Region to the right of the pole is the region of convergence. That is $\mathrm{Re}(s)=\sigma>2$.

3. Using the differentiation properties of the Laplace transform write the Laplace transform of the differential equation

$$x''(t)+5x'(t)-7x(t)=e^{-3t}u(t)$$

$$s^2X(s)-sx(0^-)-\left(\frac{d}{dt}(x(t))\right)_{t=0^-}+5\left[sX(s)-x(0^-)\right]-7X(s)=\frac{1}{s+3}\ .$$