Solution of ECE 316 Final Examination S04

1. A DT signal, $\mathrm{x}[n]$, is formed by sampling a CT sinusoid, $\mathrm{x}(t) = A\cos\left(2\pi f_o t - \frac{\pi}{3} + \theta\right)$, at exactly its Nyquist rate with one of the samples occurring exactly at time, $t = 0$.

(a) What value of $\theta_{\max}$ in the range, $-\frac{\pi}{2} \le \theta_{\max} \le \frac{\pi}{2}$, will maximize the signal power of $\mathrm{x}[n]$, and, in terms of *A*, what is that maximum signal power?

$\theta_{\max} = \frac{\pi}{3}$ Maximum signal power = A^2

Maximum power occurs when the function is a cosine with no phase shift because then all the samples are at the peaks of the cosine. This occurs when $\theta = \frac{\pi}{3}$. Then the DT signal is

$$\mathrm{x}[n] = A\cos(2\pi f_o n T_s) = A\cos\left(2\pi f_o n \frac{1}{2 f_0}\right) = A\cos(\pi n) = A(-1)^n$$

The signal power is then

$$P_x = \frac{1}{N}\sum_{n=0}^{1}\left|A(-1)^n\right|^2 = \frac{2A^2}{2} = A^2$$

(a) What value of $\theta_{\min}$ in the range, $-\frac{\pi}{2} \le \theta_{\min} \le \frac{\pi}{2}$, will minimize the signal power of $\mathrm{x}[n]$, and, in terms of *A*, what is that minimum signal power?

$\theta_{\min} = -\frac{\pi}{6}$ Minimum signal power = 0

The minimum signal power occurs when the samples all occur at zero crossings. That happens when the signal is an unshifted sine function. That occurs when $\theta = -\frac{\pi}{6}$. Then

$\mathrm{x}(t) = A\cos\left(2\pi f_o t - \frac{\pi}{3} - \frac{\pi}{6}\right) = A\sin(2\pi f_0 t)$.

2. A wagon wheel has eight spokes. It is rotating at a constant angular velocity. Four snapshots of the wheel are taken at the four times illustrated below. Let $T_s = 10\ \text{ms}$.

(a) In Case 1, what are the three lowest positive angular velocities (in revolutions per second, rps) at which it could be rotating?

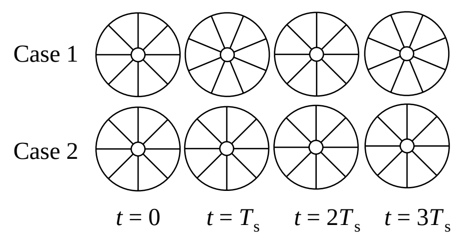
Three lowest revolutions per second = 6.25, 18.75 and 31.25 rps

The slowest possible (non-zero) rotation would be 1/16 of a revolution in one sampling time. That means one complete revolution in 160 ms which equates to a rotation rate of 6.25 rps. The next slowest rotation rate would be 3/16 of a revolution in one sampling time which equates to 18.75 rps. The next one would be 5/16 of a revolution and 31.25 rps.

(b) In Case 2, what are the three lowest positive angular velocities (in revolutions per second, rps) at which it could be rotating?

Three lowest revolutions per second = 12.5, 25 and 37.5 rps

The slowest possible (non-zero) rotation would be 1/8 of a revolution in one sampling time. That means one complete revolution in 80 ms which equates to a rotation rate of 12.5 rps. The next two lowest rates are 25 and 37.5 rps.



3. A CT signal can be simultaneously unlimited in time and frequency.

True

4. The simplest form of Shannon's sampling theorem says that it is possible to recover a signal from its samples if it is sampled at more than twice the highest frequency in the signal. In the case of narrow-band, bandlimited signals the sampling can sometimes be done at a lower frequency and the signal can still be recovered from the samples. What is the absolute lower limit on sampling rate for any signal, no matter what?

Twice the bandwidth of the signal.

5. A signal is sampled and the DFT of those samples is found to be the set of numbers, $\{X[0], X[1], X[2], \quad , X[127]\}$. How many of these numbers are guaranteed to be real numbers, no matter what the signal is, and which ones are they?

$X[0]$ and $X[64]$

6. For each of the following candidate functions indicate whether it could (Yes) or could not (No) be the autocorrelation of a real power signal and, if it could, give the numerical value of the signal power.

(a) $\mathrm{R}(\tau) = 20\cos(200\pi\tau)$ Yes Signal Power 20

(b) $\mathrm{R}(\tau) = 4\left[\mathrm{tri}\left(\dfrac{t-2}{3}\right) + \mathrm{tri}\left(\dfrac{t+2}{3}\right)\right]$ No

Maximum does not occur at $\tau = 0$.

7. If $\mathrm{x}(t) = 3e^{-2t}\cos(8t-24)\mathrm{u}(t-3) \xleftrightarrow{\mathcal{L}} \mathrm{X}(s) = A\dfrac{s+a}{s^2+bs+c}e^{ds}$, find the numerical values of *A*, *a*, *b*, *c* and *d*.

A = 3 *a* = 2 *b* = 4 *c* = 68 *d* = -3

$$3e^{-2t}\cos(8t-24)\mathrm{u}(t-3) = 3e^{-2t}\cos(8(t-3))\mathrm{u}(t-3)$$

$$3e^{-2t}\cos(8(t-3))\mathrm{u}(t-3) \xleftrightarrow{\mathcal{L}} 3\frac{s+2}{(s+2)^2+64}e^{-3s} = 3\frac{s+2}{s^2+4s+68}e^{-3s}$$

8. For each Laplace transform below indicate whether the final value theorem applies or not. If it applies, find the numerical final value.

(a) $\mathrm{X}(s) = \dfrac{s-3}{s(s+5)}$ Applies

Final Value , $\lim_{t\to\infty}\mathrm{x}(t) = \lim_{s\to 0} s\,\mathrm{X}(s) = s\dfrac{s-3}{s(s+5)} = -\dfrac{3}{5} = -0.6$

(a) $\mathrm{X}(s) = \dfrac{s^2+7}{s^2+4}$ Does Not Apply , Two poles on ω axis.

Final Value , $\lim_{t\to\infty}\mathrm{x}(t)$ = NA

9. Below are the forward path transfer function, $K\,\mathrm{H}_1(s)$, and the feedback path transfer function, $\mathrm{H}_2(s)$, for a feedback system with the conventional configuration. In each case determine whether, for any finite positive value of *K*, the feedback system will be unstable. ("Finite positive" means greater than zero but not infinite).

(a) $K\,\mathrm{H}_1(s) = K\dfrac{10}{s(s+5)}$, $\mathrm{H}_2(s) = \dfrac{s+1}{s+10}$ Will not be unstable

T has poles at 0, -5 and -10 and one finite zero at -1. In a root locus, the pole at 0 will migrate to the zero at -1 and the other two poles will collide on the real axis and then approach vertical axymptotes, never crossing into the right half-plane.

(b) $K\,\mathrm{H}_1(s) = K\dfrac{7}{(s+3)(s+11)}$, $\mathrm{H}_2(s) = \dfrac{1}{s+6}$ Will be unstable

T has poles at -3, -6 and -11 and no finite zeros. In a root locus, asymptotes will be at $\frac{\pi}{3}, \pi$ and $\frac{5\pi}{3}$. Therefore two of the root locus branches must eventually enter the right half-plane.

(c) $K\,\mathrm{H}_1(s) = K\dfrac{2s-6}{(s+9)(s+18)}$, $\mathrm{H}_2(s) = \dfrac{1}{s+1}$ Will be unstable

T has poles at -1, -9 and -18 and one finite zero at +3. In a root locus, at least one root locus branch will terminate on the zero at +3, thus entering the right half-plane.

10. What is the name given to a unity-gain feedback system whose forward path transfer function, $\mathrm{H}_1(s)$, has exactly one pole at $s=0$ and whose steady-state error to a unit-step excitation is zero? Type 1 System

11. Suppose a continuous-time LTI system has exactly two poles at a distance of one from the origin of the s plane at angles of $\pm\frac{3\pi}{4}$ radians and no finite zeros.

(a) Will its step response overshoot the final value before settling? Yes

Two complex conjugate poles indicate an underdamped system whose step response will overshoot.

(b) If the poles are moved such that the angles stay the same but the distance from the origin is increased to two, describe how the step response will change.

The ringing frequency of the overshoot will increase by a factor of two.

(c) If the poles are kept at a distance of one from the origin and are moved closer to the ω axis, but still in the left half-plane, describe how the step response will change.

The overshoot and ringing will become larger, approaching instability.

12. The z transform of $\mathrm{x}[n]=5(0.7)^{n+1}\mathrm{u}[n+1]$ can be written in the form, $\mathrm{X}(z)=A\frac{z}{z+a}$. Find the numerical values of A and a.

A = 3.5 a = -0.7

$$5(0.7)^{n+1}\mathrm{u}[n+1]\xleftrightarrow{\mathcal{Z}}5z\left(\frac{z}{z-0.7}-1\right)=5\left(\frac{z^2}{z-0.7}-z\right)=5\left(\frac{z^2-z^2+0.7z}{z-0.7}\right)=3.5\frac{z}{z-0.7}$$

Alternate Solution:

$$\mathrm{x}[n]=5(0.7)^{n+1}\mathrm{u}[n+1]=5(0.7)(0.7)^n\mathrm{u}[n+1]$$

$$\mathrm{X}(z)=\sum_{n=0}^{\infty}\mathrm{x}[n]z^{-n}=\sum_{n=0}^{\infty}5(0.7)(0.7)^n\mathrm{u}[n+1]z^{-n}=3.5\sum_{n=0}^{\infty}(0.7)^n\mathrm{u}[n]z^{-n}$$

$$\mathrm{X}(z)=3.5\frac{z}{z-0.7}$$

13. A lowpass CT filter is approximated by a DT filter. Which digital filter design technique guarantees that the response of the DT filter will be zero at $\Omega=\pm\pi$?

Bilinear

The bilinear z transform maps the entire range of CT frequencies from zero to infinity into the range, $0\le\Omega\le\pi$. Since the CT lowpass filter goes to zero at an infinite frequency, the DT filter designed using the bilinear z transform will go to zero at $\Omega=\pm\pi$.

14. Which digital filter design technique can create an unstable DT filter when approximating a stable CT filter?

Finite Difference

15. Match the pole-zero diagrams to the magnitude frequency response plots. Write the letter of the frequency response in the blank provided above the pole-zero plot. (The gain constants are not all one.)