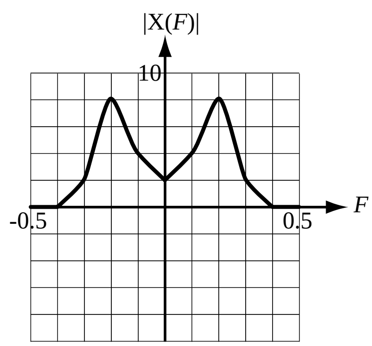
Solution to ECE 316 Test #1 Su06

1. (1 pt) Multiplying a continuous-time signal by a pulse train of the form $\mathrm{p}(t) = \mathrm{rect}(t/w) * f_s \,\mathrm{comb}(f_s t)$ is called <u>pulse amplitude modulation</u>.

2. A continuous-time signal $\mathrm{x}(t)$ is sampled above the Nyquist rate at 1000 samples/second to form a discrete-time signal $\mathrm{x}[n]$ and is also impulse sampled at the same rate to form $\mathrm{x}_\delta(t)$. The DTFT of $\mathrm{x}[n]$ is $\mathrm{X}(F)$. The CTFT of $\mathrm{x}_\delta(t)$ is $\mathrm{X}_\delta(f)$. Given the graph below answer these questions.



(a) (5 pts) What is the magnitude of $\mathrm{X}_\delta(f)$ at $f = 100$? $\mathrm{X}_\delta(100) = \underline{4}$

$\mathrm{X}_\delta(f) = \mathrm{X}(F)_{F \to f/f_s}$ and $f_s = 1000$ Therefore $|\mathrm{X}_\delta(100)| = |\mathrm{X}(0.1)| = 4$

(b) (4 pts) What is the Nyquist rate for $\mathrm{x}(t)$? $f_{NYQ} = \underline{800}$

From the graph $F_{NYQ} = 0.4 \times 2 = 0.8 \Rightarrow f_{NYQ} = f_s F_{NYQ} = 0.8 \times 1000 = 800$

3. Find the numerical values of the Nyquist rates of these signals. (If a signal is not bandlimited, just write “infinite”.)

(a) (4 pts) $\mathrm{x}(t) = 15\,\mathrm{sinc}(1000t)$ <u>1000</u>

$$\mathrm{X}(f) = (15/1000)\,\mathrm{rect}(f/1000) \Rightarrow f_m = 500 \Rightarrow f_{NYQ} = 1000$$

(b) (4 pts) $\mathrm{x}(t) = 3\sin(56\pi t) + 9\cos(28\pi t)$ <u>56</u>

Two sinusoids added with frequencies 28 and 14 Hz. Their CTFT’s simply add. Twice the highest frequency is 56 Hz.

(c) (6 pts) $\mathrm{x}(t) = -4\sin(1000\pi t)\cos(320\pi t)$ <u>1320</u>

$$X(f) = -j2[\delta(f+500) - \delta(f-500)] * (1/2)[\delta(f-160) + \delta(f+160)]$$
$$X(f) = -j[\delta(f+340) + \delta(f+660) - \delta(f-660) - \delta(f-340)]$$
$$f_m = 660 \Rightarrow f_{NYQ} = 1320$$

(d) (4 pts) $x(t) = 7\,\text{rect}(20(t-0.4))$ <u>Infinite</u>

Time-Limited, therefore Not Bandlimited

(e) (4 pts) $x(t) = 5\,\text{tri}(30t) + 8\cos(3000t)$ <u>Infinite</u>

$$X(f) = (1/6)\text{sinc}^2(f/30) + 4[\delta(f-1500) + \delta(f+1500)]$$

The sinc-squared function never goes to zero and stays there at a finite frequency.

(f) (6 pts) $x(t) = 3\text{sinc}(12t)\sin(60\pi t)$ <u>72</u>

$$X(f) = (1/4)\text{rect}(f/12) * (j/2)[\delta(f+30) - \delta(f-30)]$$
$$X(f) = (j/8)\left[\text{rect}\left(\frac{f+30}{12}\right) - \text{rect}\left(\frac{f-30}{12}\right)\right]$$

Rectangles are centered at ± 30 with full widths of 12. Therefore the highest frequency in the signal is 36.

4. A periodic signal with a fundamental frequency of 6 Hz is sampled at a rate of 36 samples/second. The sampling begins with the first sample being taken at time $t = 0$. The first 6 samples are $\{-9, 3, 2, -6, -4, 5\}$. So the first sample taken at $t = 0$ is -9, the second sample taken at $t = 1/36$ is 3, etc... If this sampling continues indefinitely what are the numerical values of

(a) (3 pts) The 9th sample? <u>2</u>

9th sample: $x(8T_s) = x(2T_s) = 2$

(b) (3 pts) The 20th sample? <u>3</u>

20th sample: $x(19T_s) = x(T_s) = 3$

(c) (4 pts) The 215th sample? <u>-4</u>

215th sample: $x(214T_s) = x((214 - 35 \times 6)T_s) = x(4T_s) = -4$

5. A set of numbers $\{x[0], x[1], x[2], x[3]\}$ is transformed using the DFT into another set of numbers $\{X[0], X[1], X[2], X[3]\}$. If $x[0] = 7$, $x[1] = 0$, $x[3] = 5$, $X[0] = 0$ and $X[1] = 2 + j12$.

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This last equation $X[1] = 2 + j12$ *is inconsistent with the other data. This is my error. Since it is inconsistent I looked for any logical analysis based on the bad data and counted any logical conclusion from it as correct. Also, if you saw that it was inconsistent and explained why, I counted that as correct, as indicated at the top of the first page of the test.*

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(a) (4 pts) What is the numerical value of $x[2]$ (not $X[2]$)? -12

$$X[0] = 0 = x[0] + x[1] + x[2] + x[3] = 7 + 0 + x[2] + 5 \Rightarrow x[2] = -12$$

If you instead used

$$X[1] = 2 + j12 = \sum_{n=0}^{3} x[n] e^{-j\pi n/2} = 7 + 0 \times (-j) + x[2] \times (-1) + 5 \times j$$

$$2 + j12 = x[2] + 7 + j5 \Rightarrow x[2] = -5 + j7$$

you get the absurd result that $x[2]$ is complex which should clue you in that there is something wrong in the statement of the problem. It is not consistent.

(b) (5 pts) What is the numerical value of $X[2]$ (not $x[2]$)? -10

$$X[k] = \sum_{n=0}^{N_F - 1} x[n] e^{-j2\pi nk/N_F} = \sum_{n=0}^{3} x[n] e^{-j\pi nk/2}$$

$$X[2] = \sum_{n=0}^{3} x[n] e^{-j\pi n} = 7 - 0 - 12 - 5 = -10$$

(c) (5 pts) What is the numerical value of $X[3]$ (not $x[3]$)? $19 - j5$

If you use $X[1] = 2 + j12$ and $X[k] = X^*[-k]$ and $X[k] = X[k + 4q]$, q an integer, you get $X[3] = 2 - j12$. If you did that and used this logic I counted it as correct.

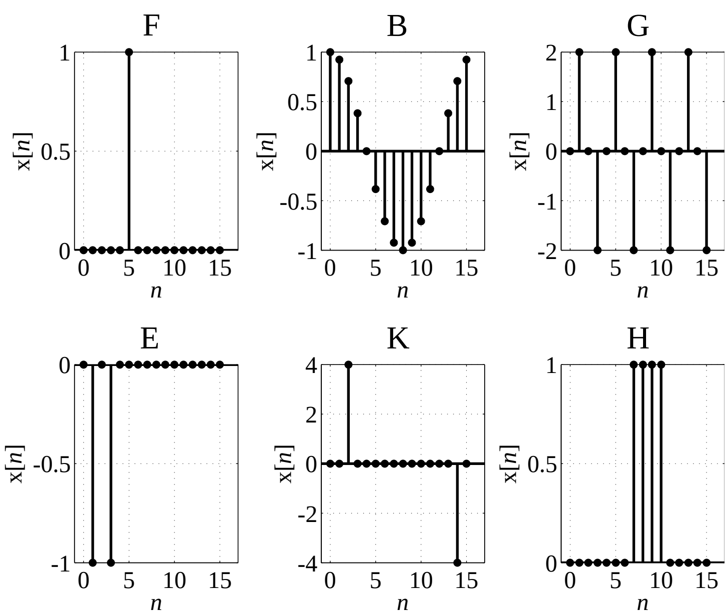
Using the correct data from above,

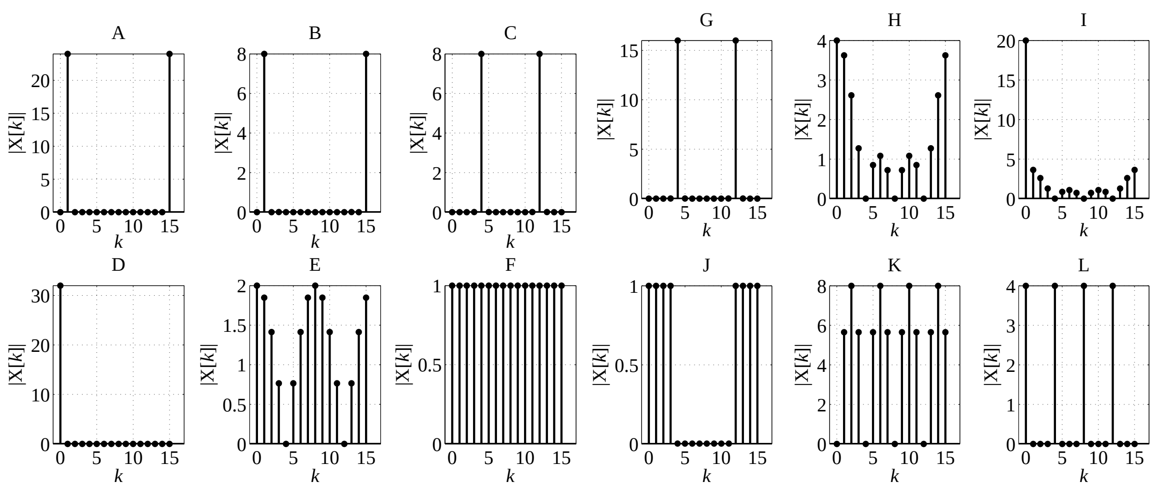
$$X[k] = \sum_{n=0}^{N_F - 1} x[n] e^{-j2\pi nk/N_F} = \sum_{n=0}^{3} x[n] e^{-j\pi nk/2}$$

$$\mathrm{X}[3] = \sum_{n=0}^{3} \mathrm{x}[n] e^{-j3\pi n/2} = 7 + 0 \times j - (-12) + 5 \times (-j) = 19 - j5 .$$

I also counted this as correct.

6. Below are some graphs of data $\{\mathrm{x}[0], \mathrm{x}[1], \quad , \mathrm{x}[15]\}$ processed by the DFT to yield sets of data $\{\mathrm{X}[0], \mathrm{X}[1], \quad , \mathrm{X}[15]\}$ whose magnitudes are graphed below the x graphs. For each x find the graph of the corresponding X and write its letter designation in the blank space provided (4 pts each).





#1 Its DFT is

$$\mathrm{X}[k] = \sum_{n=0}^{15} \delta[n-5] e^{-j2\pi nk,/16} = e^{-j10\pi k,/16}$$

and its magnitude is one for any *k*. The only possible answer is F.

#2 This is a single cosine cycle and therefore its DFT must be an impulse at $k = 1$ (and at $k = -1$ implying also at $k = 15$). There are two answers that have these characteristics so we must consider the amplitude of 1. Multiplying it by 1/2 for each impulse and then multiplying by the number of points we get $1/2 \times 16 = 8$. So the answer must be B. Also the signal power in a signal and in its DTFS are the same by Parseval's theorem. So

$$\sum_{n=0}^{15} \cos^2(2\pi n/16) = \sum_{k=0}^{15} \left|\mathrm{X}[k]/16\right|^2$$

(Division by 16 because the DTFS is the DFT, divided by the number of points, 16.) The amplitude of the cosine is 1, and the signal power of any sinusoid is half the square of its amplitude. Therefore its signal power is 1/2. In DFT B,

$$\sum_{k=0}^{15} \left|\mathrm{X}[k]/16\right|^2 = (8/16)^2 + (8/16)^2 = 1/4 + 1/4 = 1/2 .$$

#3 This is four cycles of a sine so the answer must have impulses at $k = \pm 4$, implying one also at $k = 12$ because of the period of 16. Both C and G have that property so the final choice has to be made, just as in #2, on amplitude and $2 \times 1/2 \times 16 = 16$ and G is the only correct choice. The signal power in #3 is $2^2/2 = 2$ and the signal power in G is

$$\sum_{k=0}^{15} \left|\mathrm{X}[k]/16\right|^2 = (16/16)^2 + (16/16)^2 = 1 + 1 = 2 .$$

#4 The sum of the amplitudes is -2. Therefore $\left|\mathrm{X}[0]\right| = 2$. Only E has that property.

#5 The sum of the amplitudes is 0. Therefore $\left|\mathrm{X}[0]\right| = 0$. Among the remaining graphs only K has that property.

#6 The sum of the amplitudes is 4. Therefore $\left|\mathrm{X}[0]\right| = 4$. Among the remaining graphs only H and L have that property. The signal is a rectangular pulse so the transform should be a dirichlet function (a periodically repeated sinc function). Only H has that property.