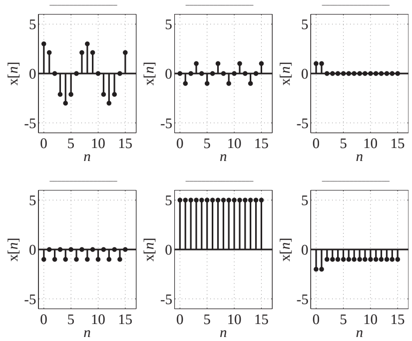
Solution of ECE 316 Test 2 Su07

1. Match the time domain graphs to the magnitude DFT graphs by writing in the letter for the correct magnitude DFT graph in the space provided.

K G D



E A I

K because it is the only 2nd harmonic sinusoid.

G because it is a 4th harmonic sinusoid and it amplitude is correct. The fourth harmonic is

$$\mathrm{X}[4] = \sum_{n=0}^{15} \mathrm{x}[n] e^{-j\pi n/2} = \sum_{n=0}^{15} \mathrm{x}[n](-j)^n$$
$$= 0 + j + 0 + j + 0 + j + 0 + j + 0 + j + 0 + j + 0 + j + 0 + j = j8$$

whose magnitude is is the sum of the magnitudes of the impulses in the sinusoid (8). It is also the number of points times the amplitude divided by two (16)(1/2).

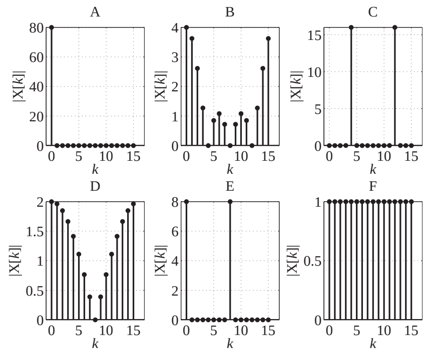
D because it is the magnitude of a cosine of amplitude 2 which is the sum of the impulse strengths in the time domain.

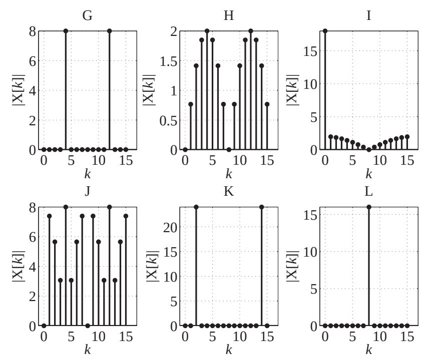
$$\mathrm{X}[k] = \sum_{n=0}^{15} \mathrm{x}[n] e^{-j\pi nk/2} = 1 + e^{-j2\pi k/16} = e^{-j\pi k/16} \times 2\cos(\pi k/16)$$
$$|\mathrm{X}[k]| = 2|\cos(\pi k/16)|$$

E because the time-domain signal is an 8th harmonic sinusoid plus a constant and zeroth harmonic is the sum of the points.

A because it is an impulse at the zeroth harmonic whose strength is the sum of the impulses in the time domain.

I because the time-domain function is the sum of a constant -1 and two impulses of strength -1 at $n=0$ and $n=1$. The zeroth harmonic is the sum of the impulses in the time domain (18) and the rest of the harmonic function is a cosine due to the two extra impulses.





2. A continuous-time sinusoid with an amplitude of one and a fundamental frequency of 24 Hz is impulse sampled at a rate of 11 samples/second. The impulse-sampled signal is passed through an ideal unity-gain lowpass filter with a bandwidth of 6 Hz. What are the amplitude and frequency of the sinusoidal output signal of the filter?

The original signal has impulses of strength 1/2 at +24 and -24. The impulse sampling creates impulses at 24 above and 24 below all integer multiples of 11 Hz. This creates an array of impulses, all of strength 11/2 at these frequencies.

f -24 +24 -13 35 -35 13 -2 46 2 -46

The only impulses that get through the filter are at +2 and -2. Therefore the frequency of the output sinusoid is 2 Hz and its amplitude is 11.

Amplitude = <u>11</u> Frequency = <u>2</u>

3. A continuous-time signal with a fundamental period of 2 seconds is sampled at a rate of 6 samples/second. Some selected values of the discrete-time signal that results are

$$x[0]=3 \ , \ x[13]=1 \ , \ x[7]=-7 \ , \ x[33]=0 \ , \ x[17]=-3$$

Find the following numerical values, if it is possible to do so. If it is impossible, explain why.

The period of the sampled signal is 12.

(a) $x[24]=x[12]=x[0]=3$.

(b) $x[18]=x[6]=x[30]=x[42]$ none of which is given.
Impossible.

(c) $x[21]=x[33]=0$

(d) $x[103]=x[103-8\times 12]=x[103-96]=x[7]=-7$

4. A continuous-time signal is sampled three times and the samples are $\mathrm{x}[0]=3$, $\mathrm{x}[1]=-1$, $\mathrm{x}[2]=7$. The DFT of this set of three samples is $\mathrm{X}[k]$. Find the numerical value of $\mathrm{X}[2]$.

$$\mathrm{X}[2]=\sum_{n=0}^{2}\mathrm{x}[n]e^{-j2\pi n(2)/3}=\sum_{n=0}^{2}\mathrm{x}[n]e^{-j4\pi n/3}$$

$$\mathrm{X}[2]=3-1\left(e^{-j4\pi/3}\right)+7\left(e^{-j8\pi/3}\right)=3+0.5-j0.866-3.5-j6.062=-j6.9282$$

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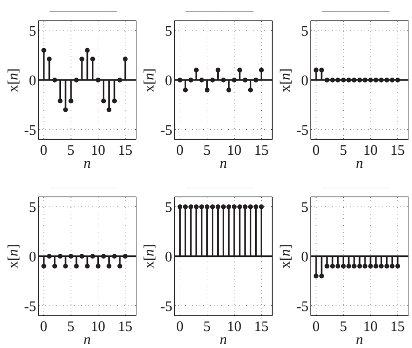
1. Match the time domain graphs to the magnitude DFT graphs by writing in the letter for the correct magnitude DFT graph in the space provided.

H J A



B D L

H because it is the only 2nd harmonic sinusoid.

J because it is a 4th harmonic sinusoid and it amplitude is correct. The fourth harmonic is

$$\mathrm{X}[4] = \sum_{n=0}^{15} \mathrm{x}[n] e^{-j\pi n/2} = \sum_{n=0}^{15} \mathrm{x}[n](-j)^n$$
$$= 0 + j + 0 + j + 0 + j + 0 + j + 0 + j + 0 + j + 0 + j + 0 + j = j8$$

whose magnitude is is the sum of the magnitudes of the impulses in the sinusoid (8). It is also the number of points times the amplitude divided by two (16)(1/2).

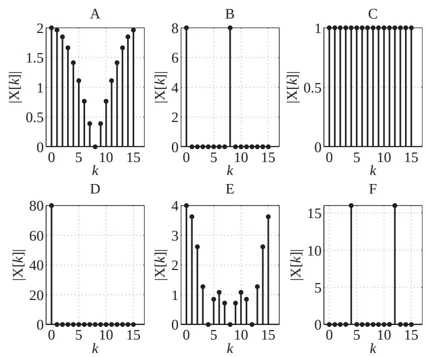
A because it is the magnitude of a cosine of amplitude 2 which is the sum of the impulse strengths in the time domain.

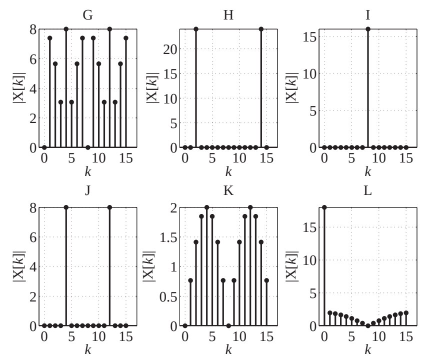
$$\mathrm{X}[k] = \sum_{n=0}^{15} \mathrm{x}[n] e^{-j\pi nk/2} = 1 + e^{-j2\pi k/16} = e^{-j\pi k/16} \times 2\cos(\pi k/16)$$
$$|\mathrm{X}[k]| = 2|\cos(\pi k/16)|$$

B because the time-domain signal is an 8th harmonic sinusoid plus a constant and zeroth harmonic is the sum of the points.

D because it is an impulse at the zeroth harmonic whose strength is the sum of the impulses in the time domain.

L because the time-domain function is the sum of a constant -1 and two impulses of strength -1 at $n = 0$ and $n = 1$. The zeroth harmonic is the sum of the impulses in the time domain (18) and the rest of the harmonic function is a cosine due to the two extra impulses.





2. A continuous-time sinusoid with an amplitude of one and a fundamental frequency of 48 Hz is impulse sampled at a rate of 22 samples/second. The impulse-sampled signal is passed through an ideal unity-gain lowpass filter with a bandwidth of 12 Hz. What are the amplitude and frequency of the sinusoidal output signal of the filter?

The original signal has impulses of strength 1/2 at +48 and -48. The impulse sampling creates impulses at 48 above and 48 below all integer multiples of 22 Hz. This creates an array of impulses, all of strength 11 at these frequencies.

f	-48	+48	-26	70	-70	26	-4	92	4	-92

The only impulses that get through the filter are at +4 and -4. Therefore the frequency of the output sinusoid is 4 Hz and its amplitude is 22.

Amplitude = <u>22</u>　　　　　　Frequency = <u>4</u>

3. A continuous-time signal with a fundamental period of 2 seconds is sampled at a rate of 6 samples/second. Some selected values of the discrete-time signal that results are

$$x[0]=5 \ , \ x[13]=1 \ , \ x[30]=-7 \ , \ x[33]=1 \ , \ x[17]=-3$$

Find the following numerical values, if it is possible to do so. If it is impossible, explain why.

The period of the sampled signal is 12.

(a) $x[24]=x[12]=x[0]=5$.

(b) $x[18]=x[30]=-7$

(c) $x[21]=x[33]=1$

(d) $x[103]=x[103-n\times 12]$ but no n can be found to provide a given value. Impossible

4. A continuous-time signal is sampled three times and the samples are $\mathrm{x}[0]=3$, $\mathrm{x}[1]=-1$, $\mathrm{x}[2]=-2$. The DFT of this set of three samples is $\mathrm{X}[k]$. Find the numerical value of $\mathrm{X}[2]$.

$$\mathrm{X}[2]=\sum_{n=0}^{2}\mathrm{x}[n]e^{-j2\pi n(2)/3}=\sum_{n=0}^{2}\mathrm{x}[n]e^{-j4\pi n/3}$$

$$\mathrm{X}[2]=3-1\left(e^{-j4\pi/3}\right)-2\left(e^{-j8\pi/3}\right)=3+0.5-j0.866+1+j1.732=4.5+j0.866$$