Solution of ECE 316 Test 2 Su08

1. The signal $x(t) = 5\,\mathrm{tri}(t-1) * \delta_2(t)$ is sampled at a rate of 8 samples/second with the first sample (sample number 1) occurring at time $t = 0$.

(a) What is the numerical value of sample number 6?

The signal is a periodic repetition of a triangle function of basewidth 2 shifted to the right by one. The period is 2.

$f_s = 8 \Rightarrow T_s = 1/8$. Sample number 6 occurs at time $t = 5T_s = 5/8$.
This time is within the original triangle so the value is

$$x(5/8) = 5\,\mathrm{tri}(5/8 - 1) = 5\,\mathrm{tri}(-3/8) = 5 \times 5/8 = 25/8 = 3.125$$

(b) What is the numerical value of sample number 63?

The samples repeat periodically with period 16 (2 seconds multiplied by 8 samples/second). So the numerical value of sample number 63 occurs at time $t = 62T_s = 62/8$ and that same numerical value occurs at time $t = (62 - 16 \times 3)T_s = 14T_s = 7/4$ which is within the original triangle. The numerical value of the sample at $t = 7/4$ is

$$x(7/4) = 5\,\mathrm{tri}(7/4 - 1) = 5\,\mathrm{tri}(3/4) = 5 \times 1/4 = 5/4 = 1.25$$

2. A signal $\mathrm{x}(t)$ is sampled 4 times to produce the signal $\mathrm{x}[n]$ and the sample values are

$$\{\mathrm{x}[0],\mathrm{x}[1],\mathrm{x}[2],\mathrm{x}[3]\} = \{7,3,-4,a\}.$$

This set of 4 numbers is the set of input data to the DFT which returns the set $\{\mathrm{X}[0],\mathrm{X}[1],\mathrm{X}[2],\mathrm{X}[3]\}$. (Be sure to notice that some x's are lower case and some X's are upper case.)

(a) What numerical value of *a* makes $\mathrm{X}[-1]$ a purely real number?

$$\mathrm{X}[k] = \sum_{n=0}^{N_F-1} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

Since the DFT is periodic with period 4,

$$\mathrm{X}[-1] = \mathrm{X}[3] = \sum_{n=0}^{3} \mathrm{x}[n] e^{-j3\pi n/2} = 7 + j3 + 4 - ja$$

Therefore $a = 3$ makes $\mathrm{X}[-1]$ purely real.

(b) Let $a = 9$. What is the numerical value of $\mathrm{X}[29]$?

$$\mathrm{X}[29] = \mathrm{X}[29 - 7 \times 4] = \mathrm{X}[1] = \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = 7 - j3 + 4 + j9 = 11 + j6$$

(c) If $\mathrm{X}[15] = 9 - j2$, what is the numerical value of $\mathrm{X}[1]$?

$$\mathrm{X}[15] = \mathrm{X}[-1] = 9 - j2 \Rightarrow \mathrm{X}[1] = \mathrm{X}^*[-1] = 9 + j2$$

3. Find the numerical Nyquist rates of the following signals. (If a signal is not

bandlimited write "infinity".)

(a) $x(t) = 11\text{tri}(t/4)\text{sinc}(t/2)$

Time limited signal. Therefore Nyquist rate is infinite.

(b) $x(t) = 11\cos(100\pi t)\sin(300\pi t)$

$$X(f) = (11/2)\left[\delta(f-50)+\delta(f+50)\right] * (j/2)\left[\delta(f+150)-\delta(f-150)\right]$$

$$X(f) = (j11/4)\left[\delta(f+100)+\delta(f+200)-\delta(f-200)-\delta(f-100)\right]$$

The maximum frequency in the signal is 200. Therefore the Nyquist rate is 400.

(c) $x(t) = \text{rect}(3t) * \delta_5(t)$

$$X(f) = (1/3)\text{sinc}(f/3)(1/5)\delta_{1/5}(f) = (1/15)\text{sinc}(f/3)\delta_{1/5}(f)$$

Signal is not bandlimited. Therefore Nyquist rate is infinite.

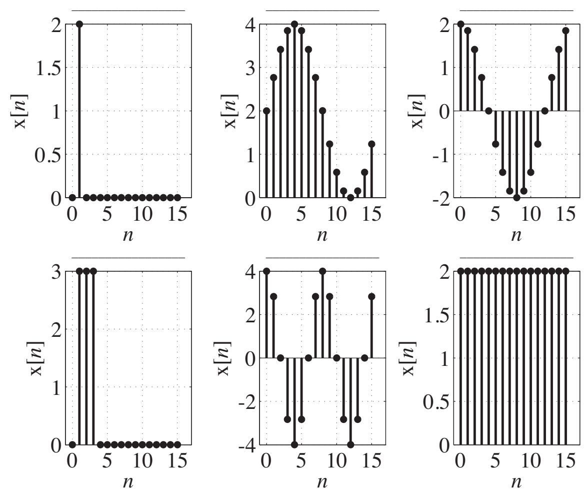
(d) $x(t) = \text{sinc}(3t) * \delta_3(t)$

$$X(f) = (1/3)\text{rect}(f/3)(1/3)\delta_{1/3}(f)$$

$$X(f) = (1/9)\text{rect}(f/3)\delta_{1/3}(f)$$

Rectangle cuts off any frequency component above 3/2. Impulses occur at integer multiples of 1/3. Therefore the highest frequency impulse not cut off by the rectangle is at 4/3. Therefore the Nyquist rate is 8/3 = 2.667.

4. In the spaces provided, associate each discrete-time signal with its corresponding DFT magnitude by writing its letter designation. (3 pts each)



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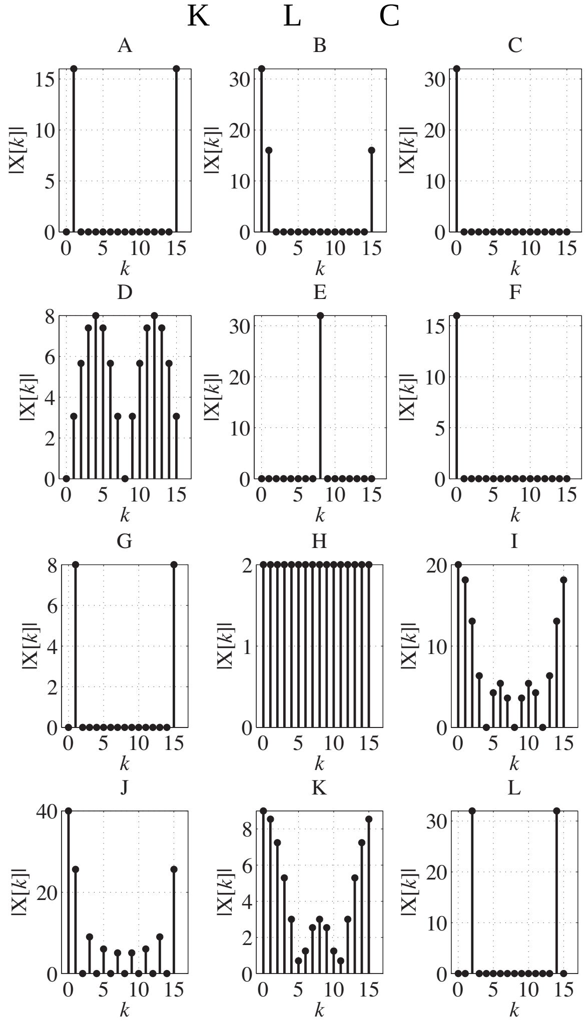


Solution of ECE 316 Test 2 Su08

1. The signal $\mathrm{x}(t) = 5\,\mathrm{tri}(t-1) * \delta_2(t)$ is sampled at a rate of 8 samples/second with the first sample (sample number 1) occurring at time $t = 0$.

(a) What is the numerical value of sample number 7?

The signal is a periodic repetition of a triangle function of basewidth 2 shifted to the right by one. The period is 2.

$f_s = 8 \Rightarrow T_s = 1/8$. Sample number 7 occurs at time $t = 6T_s = 3/4$.
This time is within the original triangle so the value is

$$\mathrm{x}(3/4) = 5\,\mathrm{tri}(3/4-1) = 5\,\mathrm{tri}(-1/4) = 5\times 3/4 = 15/4 = 3.75$$

(b) What is the numerical value of sample number 61?

The samples repeat periodically with period 16 (2 seconds multiplied by 8 samples/second). So the numerical value of sample number 61 occurs at time $t = 60T_s = 15/2$ and that same numerical value occurs at time $t = (60-16\times 3)T_s = 12T_s = 3/2$ which is within the original triangle. The numerical value of the sample at $t = 3/2$ is

$$\mathrm{x}(3/2) = 5\,\mathrm{tri}(3/2-1) = 5\,\mathrm{tri}(1/2) = 5\times 1/2 = 5/2 = 2.5$$

2. A signal $x(t)$ is sampled 4 times to produce the signal $x[n]$ and the sample values are

$$\{x[0],x[1],x[2],x[3]\} = \{7,5,-4,a\}.$$

This set of 4 numbers is the set of input data to the DFT which returns the set $\{X[0],X[1],X[2],X[3]\}$. (Be sure to notice that some x's are lower case and some X's are upper case.)

(a) What numerical value of *a* makes $X[-1]$ a purely real number?

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi kn/N_F}$$

Since the DFT is periodic with period 4,

$$X[-1] = X[3] = \sum_{n=0}^{3} x[n] e^{-j3\pi n/2} = 7 + j5 + 4 - ja$$

Therefore $a = 5$ makes $X[-1]$ purely real.

(b) Let $a = 7$. What is the numerical value of $X[29]$?

$$X[29] = X[29 - 7 \times 4] = X[1] = \sum_{n=0}^{3} x[n] e^{-j\pi n/2} = 7 - j5 + 4 + j7 = 11 + j2$$

(c) If $X[15] = 9 - j6$, what is the numerical value of $X[1]$?

$$X[15] = X[-1] = 9 - j6 \Rightarrow X[1] = X^*[-1] = 9 + j6$$

3. Find the numerical Nyquist rates of the following signals. (If a signal is not

bandlimited write "infinity".)

(a) $x(t) = 11\cos(100\pi t)\sin(500\pi t)$

$$X(f) = (11/2)[\delta(f-50)+\delta(f+50)] * (j/2)[\delta(f+250)-\delta(f-250)]$$

$$X(f) = (j11/4)[\delta(f+200)+\delta(f+300)-\delta(f-300)-\delta(f-200)]$$

The maximum frequency in the signal is 300. Therefore the Nyquist rate is 600.

(b) $x(t) = \text{rect}(3t) * \delta_5(t)$

$$X(f) = (1/3)\text{sinc}(f/3)(1/5)\delta_{1/5}(f) = (1/15)\text{sinc}(f/3)\delta_{1/5}(f)$$

Signal is not bandlimited. Therefore Nyquist rate is infinite.

(c) $x(t) = \text{sinc}(3t) * \delta_7(t)$

$$X(f) = (1/3)\text{rect}(f/3)(1/7)\delta_{1/7}(f)$$

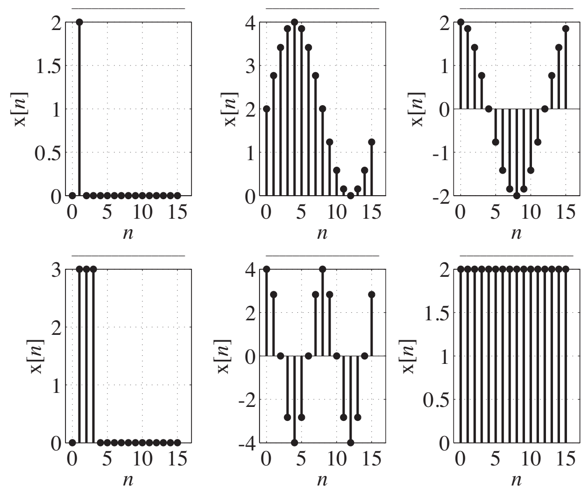
$$X(f) = (1/21)\text{rect}(f/3)\delta_{1/7}(f)$$

Rectangle cuts off any frequency component above 3/2. Impulses occur at integer multiples of 1/7. Therefore the highest frequency impulse not cut off by the rectangle is at 10/7. Therefore the Nyquist rate is 20/7 = 2.857.

(d) $x(t) = 11\,\text{tri}(t/4)\,\text{sinc}(t/2)$

Time limited signal. Therefore Nyquist rate is infinite.

4. In the spaces provided, associate each discrete-time signal with its corresponding DFT magnitude by writing its letter designation.



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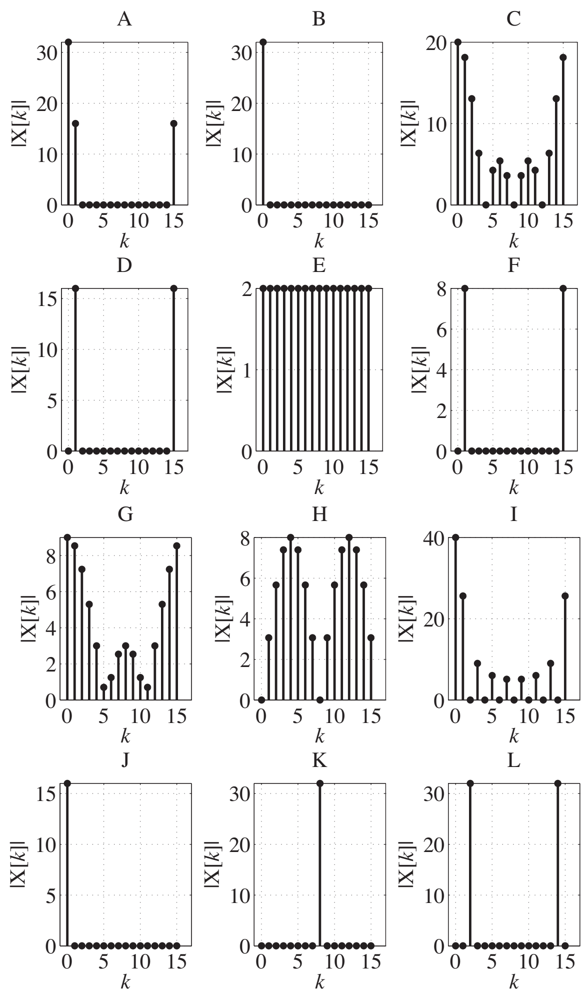


Solution of ECE 316 Test 2 Su08

1. The signal $\mathrm{x}(t) = 3\,\mathrm{tri}(t-1) * \delta_2(t)$ is sampled at a rate of 8 samples/second with the first sample (sample number 1) occurring at time $t = 0$.

(a) What is the numerical value of sample number 6?

The signal is a periodic repetition of a triangle function of basewidth 2 shifted to the right by one. The period is 2.

$f_s = 8 \Rightarrow T_s = 1/8$. Sample number 6 occurs at time $t = 5T_s = 5/8$.
This time is within the original triangle so the value is

$$\mathrm{x}(5/8) = 3\,\mathrm{tri}(5/8-1) = 3\,\mathrm{tri}(-3/8) = 3 \times 5/8 = 15/8 = 1.875$$

(b) What is the numerical value of sample number 59?

The samples repeat periodically with period 16 (2 seconds multiplied by 8 samples/second). So the numerical value of sample number 59 occurs at time $t = 58T_s = 58/8$ and that same numerical value occurs at time $t = (58 - 16 \times 3)T_s = 10T_s = 5/4$ which is within the original triangle. The numerical value of the sample at $t = 5/4$ is

$$\mathrm{x}(5/4) = 5\,\mathrm{tri}(5/4-1) = 5\,\mathrm{tri}(1/4) = 5 \times 3/4 = 15/4 = 3.75$$

2. A signal $x(t)$ is sampled 4 times to produce the signal $x[n]$ and the sample values are

$$\{x[0],x[1],x[2],x[3]\} = \{7,-2,-4,a\} .$$

This set of 4 numbers is the set of input data to the DFT which returns the set $\{X[0],X[1],X[2],X[3]\}$. (Be sure to notice that some x's are lower case and some X's are upper case.)

(a) What numerical value of *a* makes $X[-1]$ a purely real number?

$$X[k] = \sum_{n=0}^{N_F-1} x[n]e^{-j2\pi kn/N_F}$$

Since the DFT is periodic with period 4,

$$X[-1] = X[3] = \sum_{n=0}^{3} x[n]e^{-j3\pi n/2} = 7 - j2 + 4 - ja$$

Therefore $a = -2$ makes $X[-1]$ purely real.

(b) Let $a = -4$. What is the numerical value of $X[29]$?

$$X[29] = X[29-7\times 4] = X[1] = \sum_{n=0}^{3} x[n]e^{-j\pi n/2} = 7 + j2 + 4 - j4 = 11 - j2$$

(c) If $X[15] = 9 + j12$, what is the numerical value of $X[1]$?

$$X[15] = X[-1] = 9 + j12 \Rightarrow X[1] = X^*[-1] = 9 - j12$$

3. Find the numerical Nyquist rates of the following signals. (If a signal is not

bandlimited write "infinity".)

(a) $x(t) = \text{rect}(3t) * \delta_5(t)$

$$X(f) = (1/3)\text{sinc}(f/3)(1/5)\delta_{1/5}(f) = (1/15)\text{sinc}(f/3)\delta_{1/5}(f)$$

Signal is not bandlimited. Therefore Nyquist rate is infinite.

(b) $x(t) = \text{sinc}(3t) * \delta_9(t)$

$$X(f) = (1/3)\text{rect}(f/3)(1/9)\delta_{1/9}(f)$$

$$X(f) = (1/27)\text{rect}(f/3)\delta_{1/9}(f)$$

Rectangle cuts off any frequency component above 3/2. Impulses occur at integer multiples of 1/9. Therefore the highest frequency impulse not cut off by the rectangle is at 13/9. Therefore the Nyquist rate is 26/9 = 2.889.

(c) $x(t) = 11\text{tri}(t/4)\text{sinc}(t/2)$

Time limited signal. Therefore Nyquist rate is infinite.

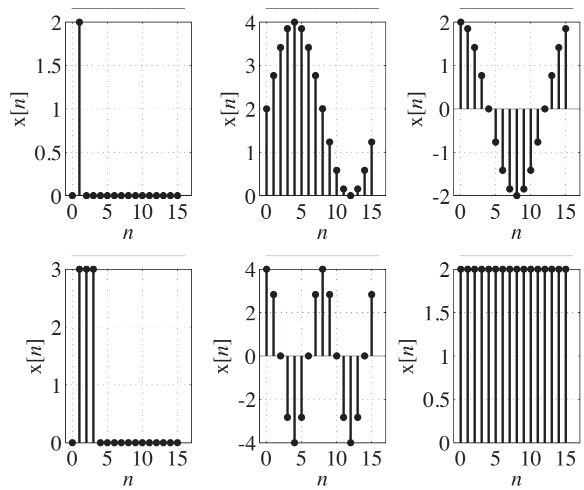
(d) $x(t) = 11\cos(40\pi t)\sin(300\pi t)$

$$X(f) = (11/2)[\delta(f-20) + \delta(f+20)] * (j/2)[\delta(f+150) - \delta(f-150)]$$

$$X(f) = (j11/4)[\delta(f+130) + \delta(f+170) - \delta(f-170) - \delta(f-130)]$$

The maximum frequency in the signal is 170. Therefore the Nyquist rate is 340.

4. In the spaces provided, associate each discrete-time signal with its corresponding DFT magnitude by writing its letter designation.



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