Solution to ECE Test #2 S09 #1

A signal $x(t)=\cos(100\pi t)$ is impulse sampled by being multiplied by the periodic impulse $\delta_{T_s}(t)$ to form the impulse-sampled signal $x_\delta(t)=\cos(100\pi t)\delta_{T_s}(t)$. Then $x_\delta(t)$ is passed through an ideal lowpass filter whose transfer function is $H(f)=\text{rect}(f/2f_c)$ to form the signal $y(t)$.

$$\left[X_\delta(f)=X(f)*(1/T_s)\delta_{1/T_s}(f)=f_s X(f)*\delta_{f_s}(f)\right] \quad , \quad y(t)\xleftrightarrow{\mathcal{F}}Y(f)$$

1. If $T_s=2$ ms and $f_c=80$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 500 Hz. The impulse-sampled signal has impulses in f at $\cdots,-550,-450,-50,50,450,550,\cdots$. The only ones that get through the filter are the ones at ± 50 Hz. So the answer is one frequency, 50 Hz.

2. If $T_s=12.5$ ms and $f_c=60$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 80 Hz. The impulse-sampled signal has impulses in f at $\cdots,-210,-110,-130,-30,-50,50,30,130,110,210,\cdots$. The only ones that get through the filter are the ones at ± 30 Hz and ± 50 Hz. So the answer is two frequencies, 30 Hz and 50 Hz.

3. If $T_s=25$ ms and $f_c=95$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 40 Hz. The impulse-sampled signal has impulses in f at $\cdots,-170,-70,-130,-30,-90,10,-50,50,-10,90,30,130,70,170\cdots$
   The only ones that get through the filter are the ones at

   $$\cdots -70,-30,-90,10,-50,50,-10,90,30,70\cdots$$

   So the answer is five frequencies, 10 Hz, 30 Hz, 50 Hz, 70 Hz and 90 Hz.

4. What is the numerical signal power of $y(t)$ in each case above?

   #1 Impulse strength is 250 which means that the sinusoid amplitude is 500 and the signal strength is $(500)^2/2=125000$

   #2 Impulse strengths are 40 which means that the sinusoid amplitudes are 80 and the signal strength is $2\times(80)^2/2=6400$

   #3 Impulse strengths are 20 which means that the sinusoid amplitudes are 40 and the signal strength is $5\times(40)^2/2=4000$

Solution to ECE Test #1 S09 #1

A signal $x(t)=\cos(120\pi t)$ is impulse sampled by being multiplied by the periodic impulse $\delta_{T_s}(t)$ to form the impulse-sampled signal $x_\delta(t)=\cos(120\pi t)\delta_{T_s}(t)$. Then $x_\delta(t)$ is passed through an ideal lowpass filter whose transfer function is $H(f)=\text{rect}(f/2f_c)$ to form the signal $y(t)$.

$$\left[X_\delta(f)=X(f)*(1/T_s)\delta_{1/T_s}(f)=f_s X(f)*\delta_{f_s}(f)\right] \quad , \quad y(t)\xleftrightarrow{\mathcal{F}}Y(f)$$

1. If $T_s=2$ ms and $f_c=80$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 500 Hz. The impulse-sampled signal has impulses in f at $\cdots,-560,-440,-60,60,440,560,\cdots$. The only ones that get through the filter are the ones at ± 60 Hz. So the answer is one frequency, 60 Hz.

2. If $T_s=12.5$ ms and $f_c=70$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 80 Hz. The impulse-sampled signal has impulses in f at $\cdots,-220,-100,-140,-20,-60,60,20,140,100,220,\cdots$. The only ones that get through the filter are the ones at ± 20 Hz and ± 60 Hz. So the answer is two frequencies, 20 Hz and 60 Hz.

3. If $T_s=20$ ms and $f_c=95$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 50 Hz. The impulse-sampled signal has impulses in f at $\cdots,-210,-90,-160,-40,-110,10,-60,60,-10,110,40,160,90,210\cdots$
   The only ones that get through the filter are the ones at

   $$\cdots -90,-60,-40,-10,10,40,60,90\cdots$$

   So the answer is four frequencies, 10 Hz, 40 Hz, 60 Hz and 90 Hz.

4. What is the numerical signal power of $y(t)$ in each case above?

   #1 Impulse strength is 250 which means that the sinusoid amplitude is 500 and the signal strength is $(500)^2/2=125000$

   #2 Impulse strengths are 40 which means that the sinusoid amplitudes are 80 and the signal strength is $2\times(80)^2/2=6400$

   #3 Impulse strengths are 25 which means that the sinusoid amplitudes are 50 and the signal strength is $4\times(50)^2/2=5000$

Solution to ECE Test #2 S09 #1

A signal $x(t)=\cos(80\pi t)$ is impulse sampled by being multiplied by the periodic impulse $\delta_{T_s}(t)$ to form the impulse-sampled signal $x_\delta(t)=\cos(80\pi t)\delta_{T_s}(t)$. Then $x_\delta(t)$ is passed through an ideal lowpass filter whose transfer function is $H(f)=\text{rect}(f/2f_c)$ to form the signal $y(t)$.

$$\left[X_\delta(f)=X(f)*(1/T_s)\delta_{1/T_s}(f)=f_s X(f)*\delta_{f_s}(f)\right] \quad , \quad y(t)\xleftrightarrow{\mathcal{F}}Y(f)$$

1. If $T_s=2$ ms and $f_c=75$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 500 Hz. The impulse-sampled signal has impulses in f at $\cdots,-540,-460,-40,40,460,540,\cdots$. The only ones that get through the filter are the ones at ± 40 Hz. So the answer is one frequency, 40 Hz.

2. If $T_s=10$ ms and $f_c=70$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 100 Hz. The impulse-sampled signal has impulses in f at $\cdots,-240,-160,-140,-60,-40,40,60,140,160,240,\cdots$. The only ones that get through the filter are the ones at ± 40 Hz and ± 60 Hz. So the answer is two frequencies, 40 Hz and 60 Hz.

3. If $T_s=20$ ms and $f_c=95$ Hz, list the numerical locations in Hz of all the impulses in $Y(f)$, in the frequency range $f\geq 0$.

   The sampling rate is 50 Hz. The impulse-sampled signal has impulses in f at $\cdots,-140,-60,-90,-10,-40,40,10,90,60,140\cdots$
   The only ones that get through the filter are the ones at

   $$\cdots -90,-60,-40,-10,10,40,60,90\cdots$$

   So the answer is four frequencies, 10 Hz, 40 Hz, 60 Hz and 90 Hz.

4. What is the numerical signal power of $y(t)$ in each case above?

   #1 Impulse strength is 250 which means that the sinusoid amplitude is 500 and the signal strength is $(500)^2/2=125000$

   #2 Impulse strengths are 50 which means that the sinusoid amplitudes are 100 and the signal strength is $2\times(100)^2/2=10000$

   #3 Impulse strengths are 25 which means that the sinusoid amplitudes are 50 and the signal strength is $4\times(50)^2/2=5000$