A bandlimited periodic CT signal, $x(t)$, whose Nyquist frequency is 25 Hz is sampled at 100 Hz over exactly one fundamental period to form the DT signal, $x[n]$. The samples are

$$\{x[0], x[1], x[2], x[3]\} = \{a, b, c, d\} \ .$$

Let one period of the DFT of those samples be $\{X[0], X[1], X[2], X[3]\}$.

(a)   What is the value of $X[1]$ in terms of a, b, c and d ?

$$X[k] = \sum_{n=0}^{N_0-1} x[n]e^{-j2\pi\frac{kn}{N_0}} \Rightarrow X[1] = \sum_{n=0}^{3} x[n]e^{-j\frac{n\pi}{2}} = a - jb - c + jd$$

(b)   What is the average value of $x(t)$ in terms of a, b, c and d ?

$$\frac{a+b+c+d}{4}$$

(c)   One of the numbers, $\{X[0], X[1], X[2], X[3]\}$, must be zero. Which one is it? $X[2]$

(d)   Two of the numbers, $\{X[0], X[1], X[2], X[3]\}$, must be real numbers. Which ones are they?

$$X[0] \ \text{and} \ X[2]$$

(e)   If $X[1] = 2 + j3$ what is the numerical value of $X[3]$? $2 - j3$