# Solution to ECE Test #1 Su05

1. Find the numerical values of the Nyquist rates of these signals. (If a signal is not bandlimited, just write "infinite".)

(a) $\mathrm{x}(t) = 15\sin(1100\pi t)$ <u>1100 Hz</u>

Highest (and only) frequency is 550 Hz.

(b) $\mathrm{x}(t) = 4\sin(28\pi t) - 6\cos(32\pi t)$ <u>32 Hz</u>

Highest frequency is 16 Hz.

(c) $\mathrm{x}(t) = -10\sin(100\pi t)\cos(420\pi t)$ <u>520 Hz</u>

$$\mathrm{X}(f) = -j5[\delta(f+50) - \delta(f-50)] * \frac{1}{2}[\delta(f-210) + \delta(f+210)]$$

$$\mathrm{X}(f) = -j(5/2)[\delta(f-160) + \delta(f+260) - \delta(f-260) - \delta(f+160)]$$

Highest frequency is 260 Hz.

(d) $\mathrm{x}(t) = 18\,\mathrm{rect}(200(t-0.02))$ <u>infinite</u>

A rectangle function is timelimited and therefore not bandlimited.

(e) $\mathrm{x}(t) = 6\,\mathrm{tri}(100t)\cos(20000\pi t)$ <u>infinite</u>

This function is timelimited and therefore not bandlimited.

(f) $\mathrm{x}(t) = 7\,\mathrm{sinc}(t/6)\sin(10\pi t)$ <u>61/6 Hz or 10.167 Hz</u>

$$\mathrm{X}(f) = 42\,\mathrm{rect}(6f) * (j/2)[\delta(f+5) - \delta(f-5)]$$

$$\mathrm{X}(f) = j21[\mathrm{rect}(6(f+5)) - \mathrm{rect}(6(f-5))]$$

Highest frequency is 5 Hz plus 1/12 Hz or 5.0833 Hz.

2. A periodic signal with a fundamental period of 10 ms is sampled at a rate of 600 samples/second. The sampling begins with the first sample being taken at time $t = 0$. The first 6 samples are $\{-3, 4, 7, -1, -8, 12\}$. So the first sample taken at $t = 0$ is -3, the second sample taken at $t = 1/600$ is 4, etc... If this sampling continues indefinitely what are the numerical values of

(a) The 8$^{th}$ sample? 4

Because of the periodicity the 8$^{th}$ sample is the same as the 2$^{nd}$ sample, 4.

(b) The 18$^{th}$ sample? 12

Because of the periodicity the 18$^{th}$ sample is the same as the 12$^{th}$ sample which is the same as the 6$^{th}$ sample, 12.

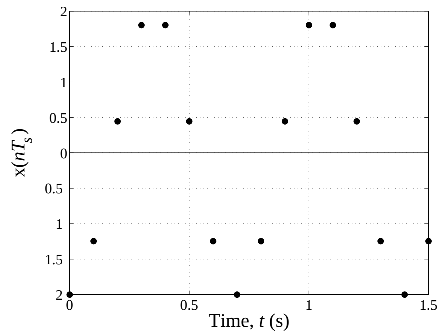
(c) The 337$^{th}$ sample? -3

$$337 / 6 = 56.166$$

$56 \times 6 = 336 \Rightarrow 336$th sample is the same as the sixth sample, 12

So the 337$^{th}$ sample is the same as the first sample, -3

3. Below is a graph of some samples taken from a sinusoid.




(a) What is the numerical sampling rate $f_s$? <u>10</u>

There are 10 samples taken in 1 second so the sampling rate is 10 samples/second or 10 Hz.

(b) If these samples have been taken properly (according to Shannon's sampling theorem) what is the numerical value of the fundamental frequency $f_0$ of the sinusoid? <u>10/7 or 1.429 Hz</u>

If the signal is oversampled its period is the same as the period of the sampled signal which is 7/10 second so $f_0 = 10/7$

(c) The sinusoid from which the samples came can be expressed in the form $A\cos(2\pi f_0 t)$. What is the numerical value of *A*? <u>-2</u>

Since there is no phase shift in $A\cos(2\pi f_0 t)$ the coefficient *A* must be negative and its magnitude is the same as the amplitude.

(d) Specify the numerical fundamental frequencies $f_{01}$ and $f_{02}$ of two other cosines of the same amplitude which, when sampled at the same rate, would yield the same set of samples. <u>11.429</u> <u>21.429</u>

Any integer multiple of the sampling rate added to the frequency of the sinusoid will create another sinusoid with the same samples. So the possible answers are $f_{01,2} = 10/7 + 10k$, *k* an integer. A few of these answers are

11.429, -8.571, 21.429, -18.571, ...

4. A set of numbers $\{x[0], x[1], x[2], x[3]\}$ is transformed using the DFT into another set of numbers $\{X[0], X[1], X[2], X[3]\}$. (Be sure to note the difference between lower-case x and upper-case X.) If $x[0]=4$, $x[1]=-1$ , $x[3]=2$, $X[0]=0$ and $X[1]=9+j3$

(a) What is the numerical value of $x[2]$? -5

$$X[k]=\sum_{n=0}^{N_F-1} x[n]e^{-j2\pi nk/N_F} \Rightarrow X[0]=\sum_{n=0}^{3} x[n]=0$$

The sum of the x values is zero because $X[0]=0$. Therefore

$$x[0]+x[1]+x[2]+x[3]=0=4-1+x[2]+2 \Rightarrow x[2]=-5$$

(b) What is the numerical value of $X[2]$ (not $x[2]$)? -2

$$X[k]=\sum_{n=0}^{N_F-1} x[n]e^{-j2\pi nk/N_F} \Rightarrow X[2]=\sum_{n=0}^{3} x[n]e^{-j\pi n}=4-(-1)+(-5)-2=-2$$

(c) What is the numerical value of $X[3]$ (not $x[3]$)? $9-j3$

Using the fact that the DFT is periodic with period 4 and $X[k]=X^*[-k]$,

$$X[3]=X[-1]=X^*[1]=(9+j3)^*=9-j3$$

5. A signal that has no signal power for all frequencies $|f|>f_m$ ($f_m$ finite) is called a bandlimited signal.

6. Filling in a signal's values between samples taken from it is called interpolation.

7. The replicas of a signal's CTFT which appear at integer multiples of the sampling rate when a signal is impulse sampled are called aliases.

8. A device whose impulse response is a causal rectangular pulse with a width equal to the time between samples can be used to approximately reconstruct a signal from its impulse samples. It is called a zero-order hold.

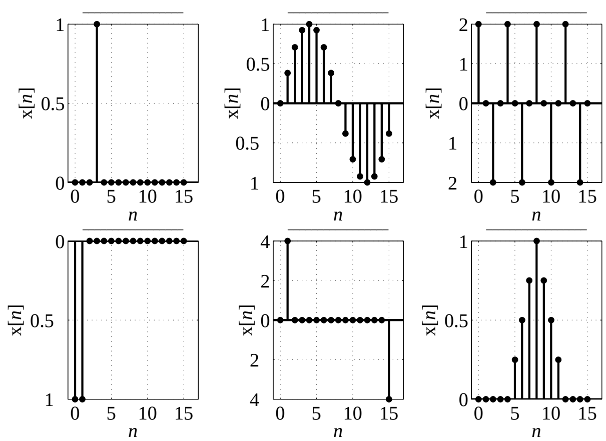
9. When a DT signal is sampled, some of the values of the original signal do not appear in the sampled signal. This effect of losing values of the original signal is called decimation.

10. Write a correct expression for the transfer function of an ideal DT lowpass filter with a cutoff frequency $F_c=0.2$.

$$\mathrm{H}(F) = \left[\mathrm{rect}(5F/2) * \mathrm{comb}(F)\right] e^{-j2\pi F n_0}$$

where $n_0$ can be any integer, including zero.

11. Below are some graphs of data $\left\{\mathrm{x}[0],\mathrm{x}[1],\ \ ,\mathrm{x}[15]\right\}$ processed by the DFT to yield sets of data $\left\{\mathrm{X}[0],\mathrm{X}[1],\ \ ,\mathrm{X}[15]\right\}$ whose magnitudes are graphed below the x graphs. For each x find the graph of the corresponding X and write its letter designation in the blank space provided.




The answers (in order) are

| | | |
|---|---|---|
| D | F | K |
| E | B | A |

Reasons:

1. A single impulse.
   Its DFT and DTFS transform magnitudes are both a constant. The only constant is D.
2. A sine wave with exactly one cycle in the representation time.
   The DFT transform magnitude is the same as the DTFS transform magntude multiplied by the number of points (16).
   The DTFS transform is
   $(j/2)\left(\mathrm{comb}_{16}[k+1] - \mathrm{comb}_{16}[k-1]\right)$. So the DFT transform is $j8\left(\mathrm{comb}_{16}[k+1] - \mathrm{comb}_{16}[k-1]\right)$. Only F has the impulses in the right place with the right strength.
3. A cosine wave with exactly four cycles in the representation time.
   The DFT transform must have an impulse at the $\pm 4$th harmonic periodically repeated with period 16. Only K has these properties.
4. Two consecutive impulses of strength -1.
   $\mathrm{X}[0]$ must be -2. Therefore $\left|\mathrm{X}[0]\right|$ must be 2. Only E has that property.

5. Impulses at 1 with strength 4 and at 15 with strength -4. Since this is one period of a periodic signal there is also an impulse at -1 with strength -4. The DTFS of those two impulses is $(j/2)\sin(\pi k/8)$ therefore the DFT of those two impulses is $j8\sin(\pi k/8)$ and its magnitude is $|8\sin(\pi k/8)|$. Only B fits this description.

6. Sampled triangle function.
The sum of all the impulses is 4 which must equal $X[0]$. Only two graphs have that property, A and G. Only A has the shape of a periodically-repeated sinc-squared function (a squared Dirichlet function). Therefore the answer is A.