

Solution of ECE 316 Test #1 Su03

1. (1 pt) When an ADC converts a CT signal to a DT signal in the form of a binary code, there is only a finite number of available unique codes it can use. Therefore the conversion is not exact and the CT sample value is only approximated by the ADC output code. The effect of representing an exact CT signal value by one of the finite number of codes is called quantization.

(I also reluctantly accepted encoding because of the way the question was worded.)

2. Find the Nyquist rates for each of these CT signals. If a signal is not bandlimited just write "infinite". (5 pts each)

(a) $x(t) = \begin{cases} |t|, & |t| < 10 \\ 0, & |t| \geq 10 \end{cases}$ Time-limited \Rightarrow Nyquist Rate = Infinite

(b) $x(t) = -8 \text{sinc}(101t) + 4 \cos(200\pi t)$ Nyquist Rate = 200

$$X(f) = \frac{-8}{101} \text{rect}\left(\frac{f}{101}\right) + 2[\delta(f-100) + \delta(f+100)]$$

Highest frequency in signal is 100 Hz.

(c) $x(t) = -32 \text{sinc}(101t) \cos(200\pi t)$ Nyquist Rate = 301

$$X(f) = \frac{-32}{101} \text{rect}\left(\frac{f}{101}\right) * \frac{1}{2}[\delta(f-100) + \delta(f+100)]$$

$$X(f) = -\frac{16}{101} \left[\text{rect}\left(\frac{f-100}{101}\right) + \text{rect}\left(\frac{f+100}{101}\right) \right]$$

$$X(f) = -\frac{16}{101} \left[\text{rect}\left(\frac{f-100}{101}\right) + \text{rect}\left(\frac{f+100}{101}\right) \right]$$

Highest frequency is 150.5 Hz.

(d) $x(t) = 7 \text{sinc}(99t) * \text{comb}(t)$ Nyquist Rate = 98

$$X(f) = \frac{7}{99} \text{rect}\left(\frac{f}{99}\right) \text{comb}(f)$$

This is a set of impulses, spaced apart by one in f . The rectangle cuts off at 49.5 Hz. Therefore the last non-zero impulse occurs at $f = 49$ and that is the highest frequency in the signal.

(e) $x(t) = 25 \text{sinc}(t) \text{comb}(100t)$ Nyquist Rate = Infinite

$$X(f) = 25 \text{rect}(f) * \frac{1}{100} \text{comb}\left(\frac{f}{100}\right) = \frac{1}{4} \text{rect}(f) * \text{comb}\left(\frac{f}{100}\right)$$

These rectangles are periodically repeated and extend to infinity in f .

3. (1 pt) Sometimes a bandpass-type signal can be adequately sampled at lower than the Nyquist rate determined by the usual statement of Shannon's sampling theorem. In such a case the absolute minimum sampling rate is greater than twice the bandwidth of the signal.

(I also accepted twice the bandwidth or the formula given in the text.)

4. (1 pt) When a CT signal is impulse sampled, scaled replicas of the CT signal's CTFT appear in the CTFT of the impulse-sampled signal at integer multiples of the sampling rate. These replicas are called aliases.

5. (5 pts) A DT signal, $x[n] = \text{sinc}\left(\frac{n}{19}\right)$, is sampled by multiplying it by $\text{comb}_{N_s}[n]$.

What is the maximum value of N_s for which the original signal can be theoretically reconstructed exactly from the samples? (Remember, N_s must be an integer.)

Maximum $N_s = \underline{18}$

The DTFT is $X(F) = 19 \text{rect}(19F) * \text{comb}(F)$. The highest DT frequency is therefore $1/38$. The DT Nyquist rate is $1/19$, a period of 19. The sampling rate must exceed this value. Since the period must be an integer, the next shorter period is 18.

6. (6 pts) An arbitrary real-valued signal is sampled 32 times to form a set of numbers, $\{x[0], x[1], \dots, x[31]\}$. These are fed into an fft algorithm on a computer and it returns the set of numbers, $\{X[0], X[1], \dots, X[31]\}$. Which of these returned numbers are guaranteed to be purely real?

$X[0]$ and $X[16]$

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi \frac{kn}{N_F}} = \sum_{n=0}^{31} x[n] e^{-j2\pi \frac{kn}{32}}$$

When $k = 0$, $X[0] = \sum_{n=0}^{31} x[n]$ and $X[0]$ is guaranteed real because all the x 's are real.

When $k = 16$, $X[16] = \sum_{n=0}^{31} x[n] e^{-j\pi n} = \sum_{n=0}^{31} x[n] (-1)^n$ and $X[16]$ is guaranteed real because all the x 's are real.

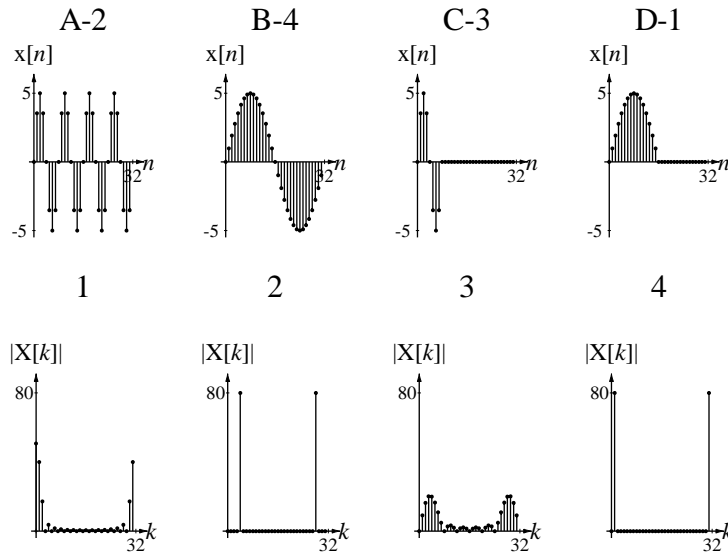
Many students, for reasons unclear to me, answered that all the even k values produced real numbers. This is easily disproved by a counterexample. Let the signal consist of a 1 followed by 31 0's. Then

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi \frac{kn}{N_F}} = e^{-j2\pi \frac{k}{32}} = e^{-j\frac{\pi k}{16}}$$

Then $X[2] = e^{-j\frac{\pi}{8}}$, $X[4] = e^{-j\frac{\pi}{4}}$, $X[6] = e^{-j\frac{3\pi}{8}}$, $X[8] = e^{-j\frac{\pi}{2}}$, ...

and none of these is purely real.

7. Below are plotted some DT signals over a range of 16 points and some 16-point DFT magnitudes. Match the DFT magnitudes to the DT signals they came from. (3 pts each)



A matches 2 because it is 4 complete cycles of a sinusoid and should appear as an impulse at the fourth harmonic number, k , $k = 4$.

B matches 4 because it is one complete cycle of a sinusoid and should appear as an impulse at the first harmonic number, k , $k = 1$.

C matches 3 because the average value is zero and 1 comes from a signal with an average value that is not zero.

D must then match 1.