



# Solution to Final Examination S05

1. A CT signal  $x(t)$  is described by  $x(t) = 4 \cos(2\pi t) \sin(20\pi t)$ .

(a) What is its numerical Nyquist rate? Nyquist Rate = 22 Hz

$$4 \cos(2\pi t) \sin(20\pi t) \xrightarrow{\mathcal{F}} j[\delta(f-1) + \delta(f+1)] * [\delta(f+10) - \delta(f-10)]$$

$$4 \cos(2\pi t) \sin(20\pi t) \xrightarrow{\mathcal{F}} j[\delta(f+9) - \delta(f-11) + \delta(f+11) - \delta(f-9)]$$

Highest frequency is 11 Hz. Therefore the Nyquist rate is 22 Hz.

(b) If  $x(t)$  is filtered by a unity-gain ideal lowpass filter with a bandwidth of 10 Hz the response is a sinusoid. What are the numerical amplitude and frequency of that sinusoid?  
Amplitude = 2      Frequency = 9 Hz

The response of the ideal lowpass filter is

$$2 \sin(18\pi t) \xrightarrow{\mathcal{F}} j[\delta(f+9) - \delta(f-9)] .$$

2. A set of samples  $\{x[0], x[1], x[2], x[3]\} = \{-9, 3, n, 8\}$  is taken from a signal and the DFT of those samples is the set of numbers  $\{X[0], X[1], X[2], X[3]\} = \{4, -11 + j5, -18, -11 - j5\}$ . What is the numerical value of  $n$ ?  
 $n = \underline{2}$

From the formula for the DFT,

$$X[0] = \sum_{n=0}^3 x[n] \Rightarrow 4 = -9 + 3 + n + 8 \Rightarrow n = 2$$

3. A CT signal has a CTFT that is zero for all  $|f| > f_m$  where  $f_m$  is finite. Based on this information alone, what word describes this signal? Bandlimited

4. When a CT signal is impulse sampled the CTFT of the impulse-sampled signal has multiple scaled replicas of the CTFT of the original CT signal which appear centered at integer multiples of the sampling rate.

(a) What are these replicas called? Aliases

(b) What is the scale factor by which the original CTFT is multiplied to yield these replicas? Scale factor =  $f_s$  (the sampling rate)

5. A CT signal has the form  $x(t) = g(t)\text{rect}(t)$  in which  $g(t)$  is finite and continuous. The Nyquist rate for  $x(t)$  is infinite. Why?

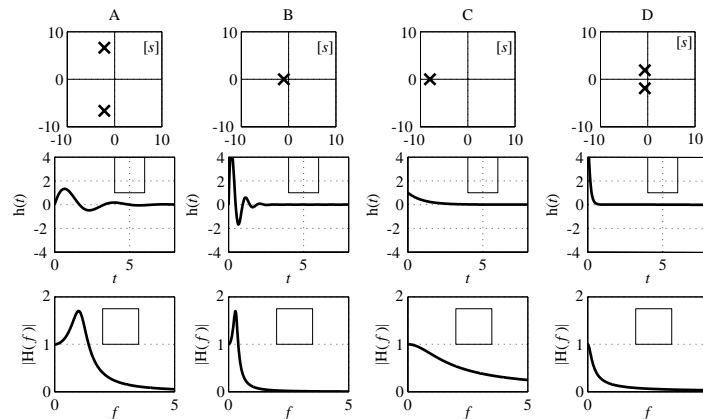
The CT signal is time-limited and cannot, therefore, be bandlimited.

6. If a CT signal can be completely described by a finite set of samples what two adjectives describe the signal? Bandlimited Periodic

7. A CT signal of the form  $x(t) = e^{-\alpha t} u(t)$ , where  $\alpha$  is a real number, has a Laplace transform but it does not have a CTFT (not even in the generalized sense). Based on this information alone, what can definitely be said about the numerical value of  $\alpha$  ?

$\alpha < 0$

8. Match the impulse-response and frequency-response plots to the pole-zero diagrams by writing the corresponding letter designation in the boxes provided.



For the impulse responses, the order is D,A,B,C.

For the frequency-response magnitudes, the order is A,D,C,B.

9. A root locus always begins on the poles of the loop transfer function and terminates on the zeros of the loop transfer function.

10. If the loop transfer function of a DT system has two finite poles and one finite zero, all located inside the unit circle of the  $z$  plane, what is true about the stability of the system as the forward-path gain  $K$  is increased?

At some finite value of  $K$  the system will become unstable.

11. A CT filter is approximated by a DT filter designed according to the direct-substitution method. If the sampling rate is 100 kHz, and a pole in the  $s$  plane at  $s = 10^5(-2 + j)$  is mapped into the  $z$  plane, find the magnitude and angle of the  $z$ -plane pole. Magnitude = 0.1353 Angle (phase) = 1 radians

$$z = e^{10^5(-2+j)/10^5} = e^{-2+j} = 0.1353e^j$$

12. In approximating a CT lowpass filter with a DT filter, which design method guarantees that the DT filter will have a zero at  $\Omega = \pm\pi$  ? Bilinear

13. If a stable CT-filter's transfer-function magnitude is zero at  $f = 0$  and approaches zero as  $f \rightarrow \infty$ , specify as completely as possible (but not more completely) everything that can logically be said about locations of the finite poles and zeros and the relationship between the number of finite poles and the number of finite zeros.

There is at least one zero at zero and the number of finite poles exceeds the number of finite zeros. All the poles are in the open left half-plane.

14. A CT system has a transfer function with a simple (single) pole at  $s = 0$  and all its other poles are in the open left half-plane.

(a) What type of signal can cause this system to have an unbounded response to a bounded excitation?

Any signal whose average value is finite but not zero.

(b) Does the final value theorem apply to this transfer function? Yes

(c) If the final-value theorem applies, is the final value of the impulse response of this system guaranteed to be zero or non-zero? Non-Zero

(d) Let this system be the forward-path of a feedback system with unity-gain feedback. If the feedback system is stable, will the response to a unit step of the feedback system approach the same value as the unit-step excitation in the limit as  $t \rightarrow \infty$  or will there be a finite difference between them in that limit?

Same Value

15. Let  $H(s) = \frac{s}{s+5}$ .

(a) Find the inverse Laplace transform  $h(t)$  of  $H(s)$ .

$$\underline{h(t) = \delta(t) - 5e^{-5t} u(t)}$$

(b) In this case  $h(t)$  is the first time derivative of a function. What is that function?

(Be precise.)  $\underline{h(t) = \frac{d}{dt}(e^{-5t} u(t))}$

16. For the unilateral Laplace transform (the one we always use), in terms of the locations of the finite poles and zeros, where is the region of convergence?

The region of convergence is the region of the  $s$  plane for which  $\sigma$  is greater than the real part of the pole with the most positive real part.

17. Which CT system-realization method starts with a cascade of integrators and the number of integrators is the same as the number of finite poles in the transfer function of the system? Canonical

18. The statement  $\alpha g[n] + \beta h[n] \xrightarrow{z} \alpha G(z) + \beta H(z)$  is the mathematical expression of one of the  $z$ -transform properties. Which one?

Linearity

19. When mapping regions of the  $s$  plane into corresponding regions of the  $z$  plane using the relationship  $z = e^{sT_s}$ , a horizontal strip extending over the range  $-\infty < \sigma < \infty$  in the  $s$  plane can map that strip exactly into the entire  $z$  plane if its width in the  $\omega$  direction is the right value. If the sampling rate is  $f_s = 20$  Hz what is the numerical value of the width of a strip in the  $\omega$  direction which just exactly maps into the entire  $z$  plane one time ?  
Width =  $40\pi$

The strip has to be wide enough that the range of  $\omega$  causes exactly one full rotation through  $2\pi$  radians in the  $z$  plane. That implies that  $\text{Width} \times T_s = 2\pi \Rightarrow \text{Width} = 2\pi f_s = 40\pi$

