General Comment: Some students indicated that if a signal is not time limited it must be bandlimited. This is not correct. A signal may be unlimited in both domains. It cannot be limited in both domains. So even if the Yes-No answer was correct the explanation was wrong in these cases.

Solution of ECE 316 Test 1 S06

Let $x(t) = 10\cos(4\pi t)$. Circle correct answers and fill in the blanks.

(a) Is $x(t)$ bandlimited? Yes

Explain your answer. The Fourier transform has impulses at ± 2 in f and is zero everywhere else.
If it is bandlimited what is its numerical Nyquist rate? 4 Hz

(b) If we multiply $x(t)$ by $\text{rect}(t)$ to form $y(t)$ is $y(t)$ bandlimited? No

Explain your answer. Multiplying it by a rectangle makes it time limited and it cannot therefore be bandlimited.
If it is bandlimited what is its numerical Nyquist rate? Not bandlimited.

(c) If we multiply $x(t)$ by $\text{sinc}(t)$ to form $y(t)$ is $y(t)$ bandlimited? Yes

Explain your answer. Fourier transform is two impulses convolved with a rectangle. Above a finite frequency magnitude the transform magnitude is zero.
If it is bandlimited what is its numerical Nyquist rate? 5 Hz

$$10\cos(4\pi t)\text{sinc}(t) \xleftrightarrow{\text{F}} 5\left[\delta(f-2)+\delta(f+2)\right] * \text{rect}(f) = 5\left[\text{rect}(f-2)+\text{rect}(f+2)\right]$$

(d) If we multiply $x(t)$ by $e^{-\pi t^2}$ to form $y(t)$ is $y(t)$ bandlimited? No

Explain your answer. Fourier transform is two impulses convolved with a Gaussian of the form $e^{-\pi f^2}$ which approaches, but never reaches, zero at a finite frequency.
If it is bandlimited what is its numerical Nyquist rate? Not bandlimited.

Solution of ECE 316 Test 1 S06

Let $x(t) = 10\cos(6\pi t)$. Circle correct answers and fill in the blanks.

(a) Is $x(t)$ bandlimited? Yes

Explain your answer. The Fourier transform has impulses at ± 3 in f and is zero everywhere else.
If it is bandlimited what is its numerical Nyquist rate? 6 Hz

(b) If we multiply $x(t)$ by $\text{sinc}(t)$ to form $y(t)$ is $y(t)$ bandlimited? Yes

Explain your answer. Fourier transform is two impulses convolved with a rectangle. Above a finite frequency magnitude the transform magnitude is zero.
If it is bandlimited what is its numerical Nyquist rate? 7 Hz

$$10\cos(6\pi t)\text{sinc}(t) \xleftrightarrow{F} 5\left[\delta(f-3)+\delta(f+3)\right] * \text{rect}(f) = 5\left[\text{rect}(f-3)+\text{rect}(f+3)\right]$$

(c) If we multiply $x(t)$ by $e^{-\pi t^2}$ to form $y(t)$ is $y(t)$ bandlimited? No

Explain your answer. Fourier transform is two impulses convolved with a Gaussian of the form $e^{-\pi f^2}$ which approaches, but never reaches, zero at a finite frequency.
If it is bandlimited what is its numerical Nyquist rate? Not bandlimited.

(d) If we multiply $x(t)$ by $\text{rect}(t)$ to form $y(t)$ is $y(t)$ bandlimited? No

Explain your answer. Multiplying it by a rectangle makes it time limited and it cannot therfore be bandlimited
If it is bandlimited what is its numerical Nyquist rate? Not bandlimited.

Solution of ECE 316 Test 1 S06

Let $x(t) = 10\cos(12\pi t)$. Circle correct answers and fill in the blanks.

(a) Is $x(t)$ bandlimited? Yes

Explain your answer. The Fourier transform has impulses at ± 6 in f and is zero everywhere else.
If it is bandlimited what is its numerical Nyquist rate? 12 Hz

(b) If we multiply $x(t)$ by $\text{rect}(t)$ to form $y(t)$ is $y(t)$ bandlimited? No

Explain your answer. Multiplying it by a rectangle makes it time limited and it cannot therefore be bandlimited
If it is bandlimited what is its numerical Nyquist rate? Not bandlimited.

(c) If we multiply $x(t)$ by $\text{sinc}(t)$ to form $y(t)$ is $y(t)$ bandlimited? Yes

Explain your answer. Fourier transform is two impulses convolved with a rectangle. Above a finite frequency magnitude the transform magnitude is zero.
If it is bandlimited what is its numerical Nyquist rate? 13 Hz

$$10\cos\left(12\pi t\right)\mathrm{sinc}\left(t\right)\overset{\mathrm{F}}{\longleftrightarrow}5\left[\delta\left(f-6\right)+\delta\left(f+6\right)\right]*\mathrm{rect}\left(f\right)=5\left[\mathrm{rect}\left(f-6\right)+\mathrm{rect}\left(f+6\right)\right]$$

(d) If we multiply $\mathrm{x}\left(t\right)$ by $e^{-\pi t^2}$ to form $\mathrm{y}\left(t\right)$ is $\mathrm{y}\left(t\right)$ bandlimited? No

Explain your answer. Fourier transform is two impulses convolved with a Gaussian of the form $e^{-\pi f^2}$ which approaches, but never reaches, zero at a finite frequency.
If it is bandlimited what is its numerical Nyquist rate? Not bandlimited.

Solution of ECE 316 Test 1 S06

Let $\mathrm{x}\left(t\right)=10\cos\left(\pi t\right)$. Circle correct answers and fill in the blanks.

(a) Is $\mathrm{x}\left(t\right)$ bandlimited? Yes

Explain your answer. The Fourier transform has impulses at $\pm 1/2$ in f and is zero everywhere else.
If it is bandlimited what is its numerical Nyquist rate? 1 Hz

(b) If we multiply $\mathrm{x}\left(t\right)$ by $\mathrm{sinc}\left(t\right)$ to form $\mathrm{y}\left(t\right)$ is $\mathrm{y}\left(t\right)$ bandlimited? Yes

Explain your answer. Fourier transform is two impulses convolved with a rectangle. Above a finite frequency magnitude the transform magnitude is zero.
If it is bandlimited what is its numerical Nyquist rate? 2 Hz

$$10\cos\left(\pi t\right)\mathrm{sinc}\left(t\right)\overset{\mathrm{F}}{\longleftrightarrow}5\left[\delta\left(f-\frac{1}{2}\right)+\delta\left(f+\frac{1}{2}\right)\right]*\mathrm{rect}\left(f\right)=5\left[\mathrm{rect}\left(f-\frac{1}{2}\right)+\mathrm{rect}\left(f+\frac{1}{2}\right)\right]$$

(c) If we multiply $\mathrm{x}\left(t\right)$ by $e^{-\pi t^2}$ to form $\mathrm{y}\left(t\right)$ is $\mathrm{y}\left(t\right)$ bandlimited? No

Explain your answer. Fourier transform is two impulses convolved with a Gaussian of the form $e^{-\pi f^2}$ which approaches, but never reaches, zero at a finite frequency.
If it is bandlimited what is its numerical Nyquist rate? Not bandlimited.

(d) If we multiply $\mathrm{x}\left(t\right)$ by $\mathrm{rect}\left(t\right)$ to form $\mathrm{y}\left(t\right)$ is $\mathrm{y}\left(t\right)$ bandlimited? No

Explain your answer. Multiplying it by a rectangle makes it time limited and it cannot therfore be bandlimited
If it is bandlimited what is its numerical Nyquist rate? Not bandlimited.