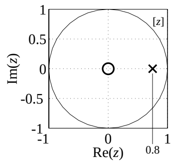
Solution of ECE 316 Test #9 S04

1. Find the magnitude of the transfer function of the systems with these pole-zero plots at the specified frequencies. (In each case assume the transfer function is of the general form, $\mathrm{H}(z) = K\frac{(z-z_1)(z-z_2)\cdots(z-z_N)}{(z-p_1)(z-p_2)\cdots(z-p_D)}$, where the z's are the zeros and the p's are the poles, and let $K=1$.)

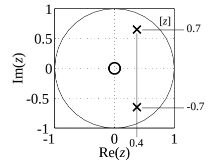
(a) $@\Omega=0$, $\left|\mathrm{H}(e^{j\Omega})\right| = \left|\mathrm{H}(1)\right| = \left|1\times\frac{1}{0.2}\right| = 5$

(b) $@\Omega=\pi$, $\left|\mathrm{H}(e^{j\Omega})\right| = \left|\mathrm{H}(-1)\right| = \left|1\times\frac{-1}{(-1-(0.4+j0.7))(-1-(0.4-j0.7))}\right| = \frac{1}{1.565\times1.565} = 0.4082$

(a)



(b)

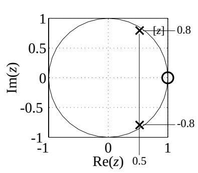


2. For each of the systems with these pole-zero plots find the DT radian frequencies, $\Omega_{\max}$ and $\Omega_{\min}$, in the range, $-\pi \le \Omega \le \pi$ for which the transfer function magnitude is a maximum and a minimum. If there is more than one value of $\Omega_{\max}$ or $\Omega_{\min}$, find all such values.

(a) The minimum magnitude obviously occurs at $z=1$ or $\Omega=0$ because the transfer function has a zero there and nowhere else. The maximum magnitude occurs when z is nearest one of the poles. That occurs where Ω is the angle formed by a vector from the origin to either one of the poles. Those Ω's are $\tan^{-1}\left(\frac{\pm0.8}{0.5}\right) = \pm1.012$.

(b) The minimum magnitude occurs at the two zeros at $\Omega = \pm\frac{\pi}{2}$. The maximum magnitude occurs at the closest approach to the pole at $z=-0.8$ which is at $\Omega=\pm\pi$.

(a) $\Omega_{\max} = \pm1.012$, $\Omega_{\min} = 0$



(b) $\Omega_{\max} = \pm\pi$, $\Omega_{\min} = \pm\frac{\pi}{2}$