Solution to ECE Test #12 S07 #1

Refer to the system diagram below with transfer function $\mathrm{H}(z) = \dfrac{\mathrm{Y}(z)}{\mathrm{X}(z)}$.

(a) Write out the transfer function in terms of the *a*'s and *b*'s. (If you don't remember, it may be helpful to think of it as $\mathrm{H}(z) = \dfrac{\mathrm{Y}_1(z)}{\mathrm{X}(z)}\dfrac{\mathrm{Y}(z)}{\mathrm{Y}_1(z)}$ and find $\dfrac{\mathrm{Y}_1(z)}{\mathrm{X}(z)}$ and $\dfrac{\mathrm{Y}(z)}{\mathrm{Y}_1(z)}$ separately.)

(b) Let $a_1 = 1.5$, $a_2 = 0.8$, $b_0 = 0$, $b_1 = 0$, $b_2 = 2$. What is the numerical value of $\mathrm{H}(1)$?

$$\mathrm{H}(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{2}{z^2 + 1.5z + 0.8} \Rightarrow \mathrm{H}(1) = \frac{2}{1 + 1.5 + 0.8} = 0.6061$$

(c) Let $a_1 = 1.5$, $a_2 = 0.8$, $b_0 = 3$, $b_1 = 0$. What numerical value of b_2 would make the transfer function have zeros on the unit circle?

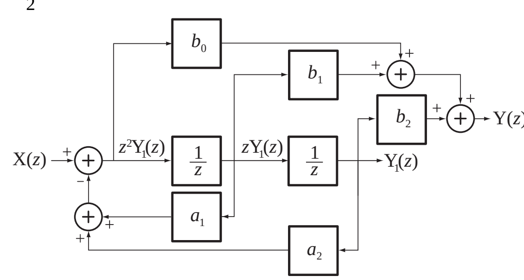
$$\mathrm{H}(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{3z^2 + b_2}{z^2 + 1.5z + 0.8}$$

For zeros on the unit circle

$$3z^2 + b_2 = 0 \Rightarrow z = \pm j\sqrt{b_2 / 3} \text{ and } |z| = 1$$

Therefore

$$\pm\sqrt{b_2 / 3} = 1 \Rightarrow b_2 = 3.$$



Solution to ECE Test #12 S07 #2

Refer to the system diagram below with transfer function $\mathrm{H}(z) = \dfrac{\mathrm{Y}(z)}{\mathrm{X}(z)}$.

(a) Write out the transfer function in terms of the a's and b's. (If you don't remember, it may be helpful to think of it as $\mathrm{H}(z) = \dfrac{\mathrm{Y}_1(z)}{\mathrm{X}(z)}\dfrac{\mathrm{Y}(z)}{\mathrm{Y}_1(z)}$ and find $\dfrac{\mathrm{Y}_1(z)}{\mathrm{X}(z)}$ and $\dfrac{\mathrm{Y}(z)}{\mathrm{Y}_1(z)}$ separately.)

(b) Let $a_1 = 1.5$, $a_2 = 0.8$, $b_0 = 0$, $b_1 = 0$, $b_2 = -3$. What is the numerical value of $\mathrm{H}(1)$?

$$\mathrm{H}(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{-3}{z^2 + 1.5z + 0.8} \Rightarrow \mathrm{H}(1) = \frac{-3}{1 + 1.5 + 0.8} = -0.9091$$

(c) Let $a_1 = 1.5$, $a_2 = 0.8$, $b_0 = 0.5$, $b_1 = 0$. What numerical value of b_2 would make the transfer function have zeros on the unit circle?

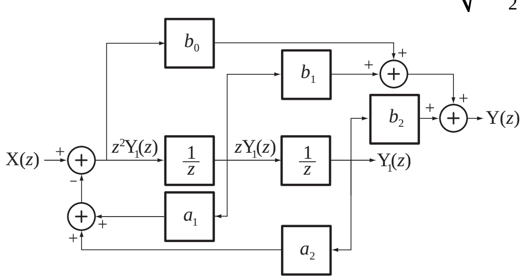
$$\mathrm{H}(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{0.5z^2 + b_2}{z^2 + 1.5z + 0.8}$$

For zeros on the unit circle

$$0.5z^2 + b_2 = 0 \Rightarrow z = \pm j\sqrt{2b_2} \quad \text{and} \quad |z| = 1$$

Therefore

$$\pm\sqrt{2b_2} = 1 \Rightarrow b_2 = 0.5.$$



Solution to ECE Test #12 S07 #3

Refer to the system diagram below with transfer function $\mathrm{H}(z) = \dfrac{\mathrm{Y}(z)}{\mathrm{X}(z)}$.

(a) Write out the transfer function in terms of the *a*'s and *b*'s. (If you don't remember, it may be helpful to think of it as $\mathrm{H}(z) = \dfrac{\mathrm{Y}_1(z)}{\mathrm{X}(z)} \dfrac{\mathrm{Y}(z)}{\mathrm{Y}_1(z)}$ and find $\dfrac{\mathrm{Y}_1(z)}{\mathrm{X}(z)}$ and $\dfrac{\mathrm{Y}(z)}{\mathrm{Y}_1(z)}$ separately.)

(b) Let $a_1 = 1.5$, $a_2 = 0.9$, $b_0 = 0$, $b_1 = 0$, $b_2 = 5$. What is the numerical value of $\mathrm{H}(1)$?

$$\mathrm{H}(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{5}{z^2 + 1.5z + 0.9} \Rightarrow \mathrm{H}(1) = \frac{5}{1 + 1.5 + 0.9} = 1.471$$

(c) Let $a_1 = 1.5$, $a_2 = 0.8$, $b_0 = 7$, $b_1 = 0$. What numerical value of b_2 would make the transfer function have zeros on the unit circle?

$$\mathrm{H}(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{7z^2 + b_2}{z^2 + 1.5z + 0.9}$$

For zeros on the unit circle

$$7z^2 + b_2 = 0 \Rightarrow z = \pm j\sqrt{b_2 / 3} \text{ and } |z| = 1$$

Therefore

$$\pm\sqrt{b_2 / 3} = 1 \Rightarrow b_2 = 7.$$