Solution of ECE 316 Test 4 Su07

1. A stable digital filter has a transfer function with one real zero and one real pole

$$\mathrm{H}(z) = \frac{z-a}{z-b} \,, \; -1 < a < 1 \,, \; -1 < b < 1 \,.$$

Its frequency response $\mathrm{H}(e^{j\Omega})$ has the following magnitudes at two discrete-time radian frequencies Ω.

Ω	0	π
$\lvert \mathrm{H}(e^{j\Omega}) \rvert$	0.3889	6.5

What are the numerical values of a and b?

a = _______________ b = _______________

$$\mathrm{H}(z) = \frac{z-a}{z-b}$$

$$\left|\mathrm{H}(e^{j0})\right| = \left|\frac{1-a}{1-b}\right| = 0.3889$$

Since both 1-a and 1-b must be positive,

$$\left|\mathrm{H}(e^{j0})\right| = \frac{1-a}{1-b} = 0.3889 \Rightarrow (1-a) = 0.3889(1-b)$$

$$a = 1 - 0.3889(1-b) = 0.6111 + 0.3889b$$

$$\left|\mathrm{H}(e^{j\pi})\right| = \left|\frac{-1-a}{-1-b}\right| = 6.5$$

Since both -1-a and -1-b must be negative,

$$\left|\mathrm{H}(e^{j\pi})\right| = \frac{-1-a}{-1-b} = 6.5 \Rightarrow (1+a) = 6.5(1+b) \Rightarrow 1.6111 + 0.3889b = 6.5 + 6.5b$$

$$6.111b = -4.889 \Rightarrow b = -0.8 \Rightarrow a = 0.3$$

a = 0.3 and b = -0.8.

2. A discrete-time feedback system has a forward-path transfer function $H_1(z) = K$ and a feedback-path transfer function $H_2(z) = 3(1+2z^{-1})$. For what range of real values of *K* is this system stable?

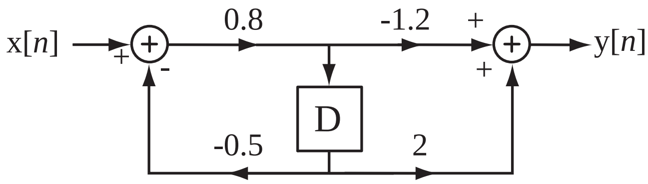
$$H(z) = \frac{K}{1+3K(1+2z^{-1})} = \frac{Kz}{z+3Kz+6K} = \frac{Kz}{z(1+3K)+6K}$$

The poles are at

$$z(1+3K)+6K = 0 \Rightarrow z = \frac{-6K}{1+3K}$$

For stability $\left|\frac{-6K}{1+3K}\right| < 1$. The transition values for *K* between stability and instability are at $\frac{-6K}{1+3K} = 1$ and at $\frac{-6K}{1+3K} = -1$ which produce *K* values of $K = 1/3$ and $-1/9$. So the *K* range for stability is $-1/9 < K < 1/3$.

3. If the system below is excited by a unit impulse, what are the numerical values of $y[0]$, $y[1]$, $y[2]$ and $y[7]$?



Let $y_1[n]$ be the output signal of the delay element. Then

$$z\,Y_1(z)/0.8 = X(z) + 0.5\,Y_1(z)$$

and

$$-1.2z\,Y_1(z) + 2\,Y_1(z) = Y(z)$$

Then

$$\frac{-1.2z+2}{1.25z-0.5}X(z) = Y(z) \Rightarrow H(z) = \frac{-1.2z+2}{1.25z-0.5} = -0.96\frac{z-1.67}{z-0.4}$$

$$h[n] = -0.96\left\{(0.4)^n u[n] - 1.67(0.4)^{n-1} u[n-1]\right\}$$

$y[0]$ = ____________ $y[1]$ = ____________

$y[2]$ = ____________ $y[7]$ = ____________

$y[0]$ = -0.96 $y[1]$ = 1.2192

$y[2]$ = 0.4877 $y[7]$ = 0.004994

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Ω	0	π
$\lvert\mathrm{H}(e^{j\Omega})\rvert$	2	0.3333

What are the numerical values of a and b?

a = _______________ b = _______________

$$\mathrm{H}(z) = \frac{z-a}{z-b}$$

$$\left|\mathrm{H}(e^{j0})\right| = \left|\frac{1-a}{1-b}\right| = 2$$

Since both 1-a and 1-b must be positive,

$$\left|\mathrm{H}(e^{j0})\right| = \frac{1-a}{1-b} = 2 \Rightarrow (1-a) = 2(1-b)$$

$$a = 1 - 2(1-b) = -1 + 2b$$

$$\left|\mathrm{H}(e^{j\pi})\right| = \left|\frac{-1-a}{-1-b}\right| = 0.3333$$

Since both -1-a and -1-b must be negative,

$$\left|\mathrm{H}(e^{j\pi})\right| = \frac{-1-a}{-1-b} = 0.3333 \Rightarrow (1+a) = 0.3333(1+b) \Rightarrow 2b = 0.3333 + 0.3333b$$

$$1.6667b = 0.3333 \Rightarrow b = 0.2 \Rightarrow a = -0.6$$

a = -0.6 and b = 0.2.

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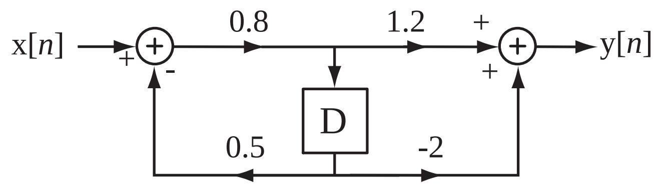

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$$h[n] = 0.96\left\{(-0.4)^n u[n] - 1.67(-0.4)^{n-1} u[n-1]\right\}$$

$y[0]$ = ____________ $y[1]$ = ____________

$y[2]$ = ____________ $y[7]$ = ____________

$y[0]$ = 0.96 $y[1]$ = -1.9872

$y[2]$ = 0.79488 $y[7]$ = -0.0081396