Solution to ECE Test #3 S08 #1

In the discrete-time filter below $a = 0.3$, $b = -2$, $c = 1.5$, $d = 0.4$

(a) The difference equation for the filter below can be expressed in the form $\mathrm{y}[n] + A\,\mathrm{y}[n-1] = B\,\mathrm{x}[n] + C\,\mathrm{x}[n-1]$. Find the numerical values of *A*, *B* and *C*.
$A =$ ________ , $B =$ ________ , $C =$ ________

$$\mathrm{y}[n] = b\left(-a\,\mathrm{x}[n] + c\,\mathrm{x}[n-1] - d\,\mathrm{y}[n-1]\right)$$

$$\mathrm{y}[n] = -2\left(-0.3\,\mathrm{x}[n] + 1.5\,\mathrm{x}[n-1] - 0.4\,\mathrm{y}[n-1]\right)$$

$$\mathrm{y}[n] - 0.8\,\mathrm{y}[n-1] = 0.6\,\mathrm{x}[n] - 3\,\mathrm{x}[n-1]$$

A = -0.8 , B = 0.6 , C = -3

(b) The frequency response $\mathrm{H}(e^{j\Omega})$ can be expressed in the form $\mathrm{H}(e^{j\Omega}) = K\dfrac{e^{j\Omega} + \alpha}{e^{j\Omega} + \beta}$.

Find the numerical values of *K*, α and β.
$K =$ ________ , $\alpha =$ ________ , $\beta =$ ________

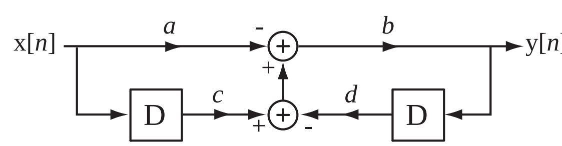
$$\mathrm{Y}(e^{j\Omega}) - 0.8\,\mathrm{Y}(e^{j\Omega})e^{-j\Omega} = 0.6\,\mathrm{X}(e^{j\Omega}) - 3\,\mathrm{X}(e^{j\Omega})e^{-j\Omega}$$

$$\mathrm{H}(e^{j\Omega}) = \frac{\mathrm{Y}(e^{j\Omega})}{\mathrm{X}(e^{j\Omega})} = \frac{0.6 - 3e^{-j\Omega}}{1 - 0.8e^{-j\Omega}} = \frac{0.6e^{j\Omega} - 3}{e^{j\Omega} - 0.8} = 0.6\frac{e^{j\Omega} - 5}{e^{j\Omega} - 0.8}$$

K = 0.6 , α = -5 , β = -0.8

(c) If $\mathrm{x}[n] = 4\,\mathrm{u}[n]$ is applied to the filter, what numerical value does $\mathrm{y}[n]$ approach as $n \to \infty$? $\left(\mathrm{y}[n]\right)_{n\to\infty} =$ ________
(You can check this answer by realizing that as $n \to \infty$, $\mathrm{y}[n]$ approaches a constant. Therefore, in that limit, $\mathrm{y}[n] = \mathrm{y}[n-1]$ and we already know that $\mathrm{x}[n] = \mathrm{x}[n-1]$. Then you can solve the difference equation for $\mathrm{y}[n]$.)

$$\mathrm{H}(e^{j0}) = \mathrm{H}(1) = 0.6\frac{1-5}{1-0.8} = 0.6\times(-4)/(0.2) = -12 \Rightarrow \mathrm{y}[n] \to 4\times(-12) = -48$$



Solution to ECE Test #3 S08 #1

In the discrete-time filter below $a = 0.5$, $b = -3$, $c = -0.2$, $d = 0.3$

(a) The difference equation for the filter below can be expressed in the form $\mathrm{y}[n] + A\mathrm{y}[n-1] = B\mathrm{x}[n] + C\mathrm{x}[n-1]$. Find the numerical values of *A*, *B* and *C*.
$A =$ ________ , $B =$ ________ , $C =$ ________

$$\mathrm{y}[n] = b\left(-a\,\mathrm{x}[n] + c\,\mathrm{x}[n-1] - d\,\mathrm{y}[n-1]\right)$$

$$\mathrm{y}[n] = -3\left(-0.5\,\mathrm{x}[n] - 0.2\,\mathrm{x}[n-1] - 0.3\,\mathrm{y}[n-1]\right)$$

$$\mathrm{y}[n] - 0.9\,\mathrm{y}[n-1] = 1.5\,\mathrm{x}[n] + 0.6\,\mathrm{x}[n-1]$$

A = -0.9 , B = 1.5 , C = 0.6

(b) The frequency response $\mathrm{H}\left(e^{j\Omega}\right)$ can be expressed in the form $\mathrm{H}\left(e^{j\Omega}\right) = K\dfrac{e^{j\Omega} + \alpha}{e^{j\Omega} + \beta}$.

Find the numerical values of *K*, α and β.

$K =$ ________ , $\alpha =$ ________ , $\beta =$ ________

$$\mathrm{Y}\left(e^{j\Omega}\right) - 0.9\,\mathrm{Y}\left(e^{j\Omega}\right)e^{-j\Omega} = 1.5\,\mathrm{X}\left(e^{j\Omega}\right) + 0.6\,\mathrm{X}\left(e^{j\Omega}\right)e^{-j\Omega}$$

$$\mathrm{H}\left(e^{j\Omega}\right) = \frac{\mathrm{Y}\left(e^{j\Omega}\right)}{\mathrm{X}\left(e^{j\Omega}\right)} = \frac{1.5 + 0.6e^{-j\Omega}}{1 - 0.9e^{-j\Omega}} = \frac{1.5e^{j\Omega} + 0.6}{e^{j\Omega} - 0.9} = 1.5\frac{e^{j\Omega} + 0.4}{e^{j\Omega} - 0.9}$$

K = 1.5 , α = 0.4 , β = -0.9

(c) If $\mathrm{x}[n] = 4\,\mathrm{u}[n]$ is applied to the filter, what numerical value does $\mathrm{y}[n]$ approach as $n \to \infty$? $\left(\mathrm{y}[n]\right)_{n\to\infty} =$ ________

(You can check this answer by realizing that as $n \to \infty$, $\mathrm{y}[n]$ approaches a constant. Therefore, in that limit, $\mathrm{y}[n] = \mathrm{y}[n-1]$ and we already know that $\mathrm{x}[n] = \mathrm{x}[n-1]$. Then you can solve the difference equation for $\mathrm{y}[n]$.)

$$\mathrm{H}\left(e^{j0}\right) = \mathrm{H}(1) = 1.5\frac{1 + 0.4}{1 - 0.9} = 1.5 \times 1.4 / 0.1 = 21 \Rightarrow \mathrm{y}[n] \to 4 \times 21 = 84$$