Solution to ECE Test #3 S09 #1

1. A system has a transfer function $\mathrm{H}(z) = \frac{2z-1.5}{3z-2}$. What is the numerical magnitude of its frequency response at

   (a) $\Omega = 0$?

   (b) $\Omega = \pi$?

   (c) $\Omega = \pi/2$?

$$\mathrm{H}(e^{j\Omega}) = \frac{2e^{j\Omega}-1.5}{3e^{j\Omega}-2} \Rightarrow \left|\mathrm{H}(e^{j0})\right| = \left|\frac{2-1.5}{3-2}\right| = 0.5$$

$$\left|\mathrm{H}(e^{j\pi})\right| = \left|\frac{-2-1.5}{-3-2}\right| = 0.7$$

$$\left|\mathrm{H}(e^{j\pi/2})\right| = \left|\frac{j2-1.5}{j3-2}\right| = \sqrt{\frac{2^2+(1.5)^2}{3^2+2^2}} = \sqrt{6.25/13} = 0.6934$$

2. For each system transfer function below, which type of ideal filter does it most closely approximate, lowpass (LP), highpass (HP), bandpass (BP) or bandstop (BS)? Circle the correct answer. (Suggestion: Look at the magnitude frequency response at $\Omega = 0$, $\pi/2$ and π.)

   (a) $\mathrm{H}(z) = \frac{z-1}{z-0.9}$ LP HP BP BS

   At $\Omega = 0$ the response magnitude is zero. At $\Omega = \pi$ the response magnitude is $2/1.9 = 1.053$. At $\Omega = \pi/2$ the response magnitude is $\left|\mathrm{H}(e^{j\Omega})\right| = \left|\frac{j-1}{j-0.9}\right| = 1.051$. Highpass.

   (b) $\mathrm{H}(z) = \frac{z^2-1}{z^2+0.8}$ LP HP BP BS

   At $\Omega = 0$ the response magnitude is zero. At $\Omega = \pi$ the response magnitude is again zero. At $\Omega = \pi/2$ the response magnitude is $\left|\mathrm{H}(e^{j\Omega})\right| = \left|\frac{j^2-1}{j^2+0.8}\right| = \left|\frac{-2}{-0.2}\right| = 10$. Bandpass.

3. For the system frequency response $\mathrm{H}(e^{j\Omega}) = \frac{e^{-jA\Omega}}{1-0.8e^{-j\Omega}}$, what numerical range of integer values of A will produce a causal system?

$$\mathrm{h}[n] = 0.8^{n-A}\,\mathrm{u}[n-A] \qquad \text{For causality, } A \ge 0.$$

Solution to ECE Test #3 S09

1. A system has a transfer function $\mathrm{H}(z) = \dfrac{3z - 1.5}{7z - 2}$. What is the numerical magnitude of its frequency response at

   (a) $\Omega = 0$?

   (b) $\Omega = \pi$?

   (c) $\Omega = \pi/2$?

$$\mathrm{H}(e^{j\Omega}) = \frac{3e^{j\Omega} - 1.5}{7e^{j\Omega} - 2} \Rightarrow \left|\mathrm{H}(e^{j0})\right| = \left|\frac{3 - 1.5}{7 - 2}\right| = 0.3$$

$$\left|\mathrm{H}(e^{j\pi})\right| = \left|\frac{-3 - 1.5}{-7 - 2}\right| = 0.5$$

$$\left|\mathrm{H}(e^{j\pi/2})\right| = \left|\frac{j3 - 1.5}{j7 - 2}\right| = \sqrt{\frac{3^2 + (1.5)^2}{7^2 + 2^2}} = \sqrt{11.25/53} = 0.4609$$

2. For each system transfer function below, which type of ideal filter does it most closely approximate, lowpass (LP), highpass (HP), bandpass (BP) or bandstop (BS)? Circle the correct answer. (Suggestion: Look at the magnitude frequency response at $\Omega = 0,\ \pi/2$ and π.)

   (a) $\mathrm{H}(z) = \dfrac{z}{z - 0.9}$ LP HP BP BS

At $\Omega = 0$ the response magnitude is 10. At $\Omega = \pi$ the response magnitude is 0.526. At $\Omega = \pi/2$ the response magnitude is $\left|\mathrm{H}(e^{j\Omega})\right| = \left|\dfrac{j}{j - 0.9}\right| = 0.7433$. Lowpass.

   (b) $\mathrm{H}(z) = \dfrac{z^2 + 1}{z^2 - 0.8}$ LP HP BP BS

At $\Omega = 0$ the response magnitude is 20. At $\Omega = \pi$ the response magnitude is again 20. At $\Omega = \pi/2$ the response magnitude is $\left|\mathrm{H}(e^{j\Omega})\right| = \left|\dfrac{j^2 + 1}{j^2 - 0.8}\right| = \left|\dfrac{0}{-1.8}\right| = 0$. Bandstop.

3. For the system frequency response $\mathrm{H}(e^{j\Omega}) = \dfrac{e^{-jA\Omega}}{1 - 0.8e^{-j\Omega}}$, what numerical range of integer values of A will produce a causal system?

$$\mathrm{h}[n] = 0.8^{n-A}\,\mathrm{u}[n - A] \qquad \text{For causality, } A \ge 0.$$