

Solution of ECE 316 Test #4 Su03 #1

1. (2 pts) The Laplace transform of a CT signal is related to the z transform of a DT signal, which was formed by sampling that CT signal, through $z = e^{sT_s}$ where T_s is the time between samples. Therefore movement along the ω axis in the s plane corresponds to movement along the unit circle in the z plane. Also the left half of the s plane corresponds to the interior of the unit circle in the z plane.

2. (5 pts) A DT signal, $x[n]$, has a z transform, $X(z) = \frac{5z^2 - z + 3}{2z^2 - \frac{1}{2}z + \frac{1}{4}}$. What is the

numerical value of $x[0]$? $x[0] = 2.5$

From the initial-value theorem,

$$g[0] = \lim_{z \rightarrow \infty} G(z)$$

$$x[0] = \lim_{z \rightarrow \infty} \frac{5z^2 - z + 3}{2z^2 - \frac{1}{2}z + \frac{1}{4}} = \frac{5}{2} = 2.5$$

3. (10 pts) A DT signal, $y[n]$, is related to another DT signal, $x[n]$, by

$$y[n] = \sum_{m=0}^n x[m] .$$

If $y[n] \xrightarrow{z} \frac{1}{(z-1)^2}$, what are the numerical values of $x[-1]$, $x[0]$, $x[1]$ and $x[2]$?

$$x[-1] = 0 \quad x[0] = 0 \quad x[1] = 0 \quad x[2] = 1$$

If $y[n] = \sum_{m=0}^n x[m]$, then, using the accumulation property of the z transform,

$$Y(z) = \frac{z}{z-1} X(z) = \frac{1}{(z-1)^2}$$

Solving for $X(z)$,

$$X(z) = \frac{z^{-1}}{z-1} = z^{-2} \frac{z}{z-1}$$

Taking the inverse transform,

$$x[n] = u[n-2] .$$

Then $x[-1] = 0$, $x[0] = 0$, $x[1] = 0$, $x[2] = 1$.

4. (10 pts) Using the z -transform tables for sine and cosine, the inverse z transform of $X(z) = 2 \frac{z^2 - 0.1488z}{z^2 - 1.75z + 1}$ can be found as a sum of a cosine and a sine of the form, $x[n] = [A \cos(\Omega_0 n) + B \sin(\Omega_0 n)]u[n]$. Find real numerical values for A , B and Ω_0 .

$$X(z) = 2 \left[\frac{z^2 - 0.875z}{z^2 - 1.75z + 1} - \frac{0.1488z - 0.875z}{z^2 - 1.75z + 1} \right] \Rightarrow \Omega_0 = 0.5054$$

$$X(z) = 2 \left[\frac{z^2 - 0.875z}{z^2 - 1.75z + 1} + \frac{0.7262z}{z^2 - 1.75z + 1} \right] = 2 \left[\frac{z^2 - 0.875z}{z^2 - 1.75z + 1} + \frac{0.7262}{0.4841} \frac{0.4841z}{z^2 - 1.75z + 1} \right]$$

$$X(z) = 2 \frac{z^2 - 0.875z}{z^2 - 1.75z + 1} + 3 \frac{0.4841z}{z^2 - 1.75z + 1}$$

$$x[n] = [2 \cos(0.5054n) + 3 \sin(0.5054n)]u[n]$$

$$A = 2 \quad B = 3 \quad \Omega_0 = 0.5054$$

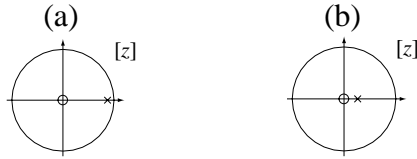
5. Classify the DT systems with these transfer functions as stable (S), marginally stable (MS) or unstable without being marginally stable (US).

(a) (4 pts) $H(z) = \frac{z-1}{(z^2 + \sqrt{2}z + 1)^2}$ Repeated poles on unit circle - US

(b) (4 pts) $H(z) = \frac{z+2}{\left(z + \frac{1}{2}\right) \left(z - e^{j\frac{\pi}{3}}\right) \left(z - e^{-j\frac{\pi}{3}}\right)}$ Simple poles on the unit circle - MS

(c) (3 pts) $H(z) = \frac{z}{(z-0.8+j0.8)(z-0.8-j0.8)}$, Poles outside the unit circle - US

6. (10 pts) Below are the pole zero diagrams for two one-pole systems. Assuming both transfer functions are of the form, $H(z) = (1-p)\frac{z}{z-p}$, where p is the pole in each case, which system, (a) or (b), will have a faster unit-sequence response. (“Faster” here means approaching its final value faster.) Circle the correct answer.



Check your answer by comparing the unit-sequence responses for the two cases to see how the pole location affects the speed of response.

(b) has the faster response

The form of the unit sequence response is

$$h_{-1}[n] \xleftarrow{z} (1-p)\frac{z}{z-1} \frac{z}{z-p} = z \left(\frac{1}{z-1} + \frac{p}{z-p} \right) = \frac{z}{z-1} - \frac{pz}{z-p}$$

$$h_{-1}[n] = (1-p^{n+1})u[n]$$

A smaller magnitude of p will cause the term, p^{n+1} , to approach zero faster, making the overall response’s approach to its final value faster.

7. (4 pts each) Using the following definitions of lowpass (LP), highpass (HP), bandpass (BP) and bandstop (BS), classify the DT systems with the following pole-zero diagrams as LP, HP, BP, BS or U for unclassifiable if a system does not meet any of the tests for LP, HP, BP and BS. In each case the transfer function is $H(z)$.

LP: $H(1) \neq 0$ and $H(-1) = 0$

HP: $H(1) = 0$ and $H(-1) \neq 0$

BP: $H(1) = 0$ and $H(-1) = 0$ and $H(z) \neq 0$ for some range of $|z|=1$

BS: $H(1) \neq 0$ and $H(-1) \neq 0$ and $H(z) = 0$ for at least one $|z|=1$

