Solution to ECE Test #4 Su05

1. Find the numerical values of the constants.

(a) $$\left(\delta[n]-2\delta[n-2]\right)*(0.7)^n\,\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}}A\frac{z+Bz^{-1}}{z+C}$$

A = 1 , B = -2 , C = -0.7

$$\left(\delta[n]-2\delta[n-2]\right)*(0.7)^n\,\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}}\left(1-2z^{-2}\right)\frac{z}{z-0.7}=\frac{z-2z^{-1}}{z-0.7}$$

(b) $$Aa^n\left[\cos(bn)+B\sin(bn)\right]\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}}\frac{z^2}{z^2+z+0.8}$$

A = 1 , B = -0.6742

a = 0.8944 , b = 2.164

$$\alpha=\sqrt{0.8}=0.8944$$

and $-2\alpha\cos(\Omega_0)=1\Rightarrow\cos(\Omega_0)=-\dfrac{1}{2\times0.8944}=-0.559\Rightarrow\Omega_0=2.164$

$$Aa^n\left[\cos(bn)+B\sin(bn)\right]\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}}\frac{z^2-0.8944\cos(2.164)}{z^2+z+0.8}+\frac{0.8944\cos(2.164)}{z^2+z+0.8}$$

$$Aa^n\left[\cos(bn)+B\sin(bn)\right]\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}}\frac{z^2+0.5}{z^2+z+0.8}-\frac{0.5}{0.8944\sin(2.164)}\frac{0.8944\sin(2.164)}{z^2+z+0.8}$$

$$Aa^n\left[\cos(bn)+B\sin(bn)\right]\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}}\frac{z^2+0.5}{z^2+z+0.8}-0.6742\frac{0.7416}{z^2+z+0.8}$$

$$(0.8944)^n\left[\cos(2.164n)+0.6472\sin(2.164n)\right]\mathrm{u}[n]\xleftrightarrow{\mathcal{Z}}\frac{z^2+0.5}{z^2+z+0.8}-0.6742\frac{0.7416}{z^2+z+0.8}$$

(c) $4\,\mathrm{u}[n+1] \xleftrightarrow{\mathcal{Z}} \dfrac{Az}{z-B}$

$A = \underline{4}$, $B = \underline{1}$

Using $\mathrm{g}[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}\left(\mathrm{G}(z) - \sum_{m=0}^{n_0-1} \mathrm{g}[m] z^{-m}\right)$, $n_0 > 0$

$$4\,\mathrm{u}[n+1] \xleftrightarrow{\mathcal{Z}} 4z\left(\frac{z}{z-1} - 1\right) = 4z\frac{z-z+1}{z-1} = \frac{4z}{z-1}$$

2. If $\mathrm{X}(z) = \dfrac{z^3 + 2z^2 - 3z + 7}{(z-1)(z^2 - 1.8z + 0.9)}$ what is the numerical final value of $\mathrm{x}[n]$ ($\lim_{n\to\infty} \mathrm{x}[n]$) ?

All the poles of $(z-1)\mathrm{X}(z)$ are in the open interior of the unit circle. Therefore the final-value theorem applies.

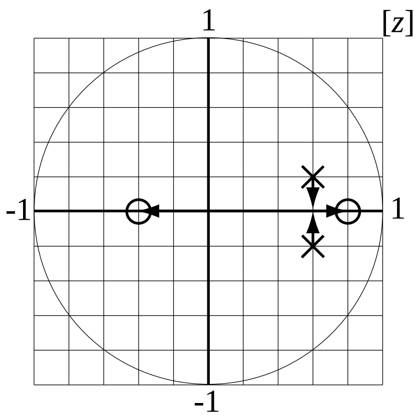
$$\lim_{n\to\infty} \mathrm{x}[n] = \lim_{z\to 1}(z-1)\mathrm{X}(z) = \lim_{z\to 1}\frac{z^3 + 2z^2 - 3z + 7}{z^2 - 1.8z + 0.9} = \frac{1+2-3+7}{1-1.8+0.9} = 70$$

3. If $(1.1)^n \cos(2\pi n/16) \xleftrightarrow{\mathcal{Z}} \mathrm{H}_1(z)$, and $\mathrm{H}_2(z) = \mathrm{H}_1(az)$ and $\mathrm{H}_1(z)$ and $\mathrm{H}_2(z)$ are transfer functions of DT systems #1 and #2 respectively, what range of values of a will make system #2 stable?

Poles of $\mathrm{H}_1(z)$ are outside the unit circle and the impulse response grows because of the factor $(1.1)^n$. If $z \to az$, then by the change of scale property, the time-domain function will be multiplied by $(1/a)^n$. If the magnitude of a is greater than 1.1, then the poles of $\mathrm{H}_2(z)$ will be inside the unit circle and the factor $(1.1)^n$ changes to $(1.1/a)^n$ which, for $|a| > 1.1$ makes the impulse response decay instead of growing.

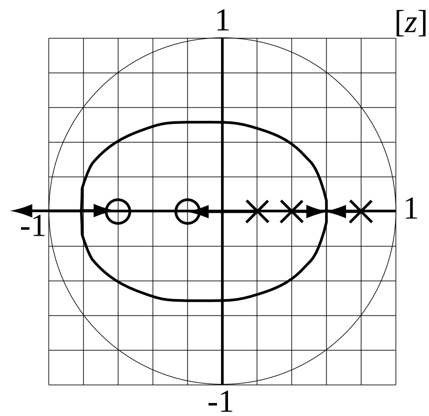
By the way, if $\mathrm{H}_2(z)$ is to be a physically realizable system then a must also be a real number. That is, $a > 1.1$ or $a < -1.1$.

4. Sketch a root locus for each pole-zero map of a loop transfer function below. Then, for each one, indicate whether the system is unstable at a finite, positive value of the gain constant *K*.



Unstable at a finite, positive *K*?

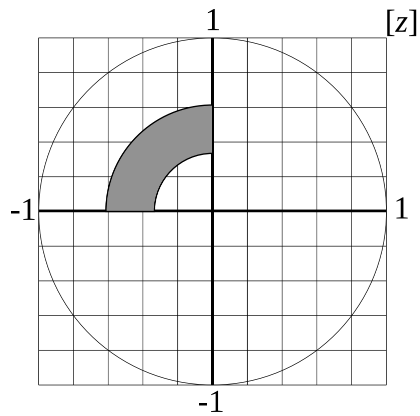
Yes **No**



Unstable at a finite, positive *K*?

Yes No

5. In the space provided below sketch the area of the z plane corresponding to the area of the s plane defined by $-\frac{1}{T_s} < \sigma < -\frac{1}{2T_s}$ and $\frac{\pi}{2T_s} < \omega < \frac{\pi}{T_s}$.



The area is defined in polar coordinates as a region for which the distance from the origin is between e^{-1} and $e^{-1/2}$ or between 0.368 and 0.6065 and for which the angle is between $\pi/2$ and π.

6. A DT system has a transfer function of the form

$$\mathrm{H}(z) = A\frac{(z - z_1)(z - z_2)}{(z - p_1)(z - p_2)} \ .$$

If $A = 2$, $z_1 = 0$, $z_2 = 0.1$, $p_1 = -0.8$ and $p_2 = -0.5$

(a) At what numerical value of Ω will the transfer function magnitude be largest?

The zeros are at 0 and 0.1. So when the vectors to the operating frequency on the unit circle rotate one has a constant length and the other has a length that changes very little. The poles are at -0.8 and -0.5. So when the operating frequency is nearest these values the pole vectors are the shortest and the transfer function magnitude is the largest. This occurs when $\Omega = \pi + 2n\pi$ where n is any integer.

(b) At what numerical value of Ω will the transfer function magnitude be smallest?

Conversely the pole vectors will be at maximum length when $\Omega = 0 + 2n\pi$ where n is any integer and the transfer function magnitude will be minimum there.