# Solution to ECE Test #4 Su06

1. If $\mathrm{x}[n] \xleftrightarrow{\mathcal{Z}} \mathrm{X}(z) = \dfrac{z}{z^2 + 0.81}$ fill in the table below with numbers.

$$\alpha^n \sin(\Omega_0 n)\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{\alpha z \sin(\Omega_0)}{z^2 - 2\alpha z \cos(\Omega_0) + \alpha^2}$$

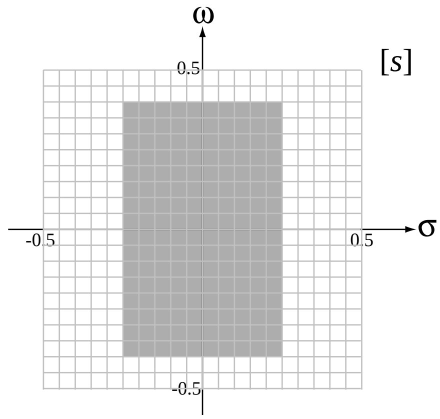
$$\alpha = \sqrt{0.81} = 0.9 \quad , \quad 2\alpha \cos(\Omega_0) = 0 \Rightarrow \Omega_0 = \pi / 2$$

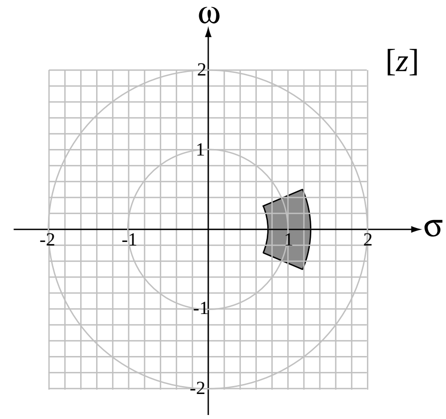
$$\mathrm{X}(z) = \frac{1}{\alpha \sin(\Omega_0)} \frac{z\alpha \sin(\Omega_0)}{z^2 + 0.81} \Rightarrow \mathrm{x}[n] = \frac{1}{\alpha \sin(\Omega_0)} \alpha^n \sin(\Omega_0 n)\mathrm{u}[n]$$

$$\mathrm{x}[n] = (0.9)^{n-1} \sin(\pi n / 2)\mathrm{u}[n]$$

| $n$ | $\mathrm{x}[n]$ |
|---|---|
| 1 | 1 |
| 3 | –0.81 |
| 11 | –0.3487 |
| 21 | 0.1216 |

2. On the right-hand graph coordinates, sketch the region in the $z$ plane corresponding to the given shaded region in the $s$ plane, using the mapping relationship $z = e^{sT_s}$ with $T_s = 1$.




The range of magnitudes of $z$ will be $e^{-0.25} = 0.7788$ to $e^{+0.25} = 1.284$. The range of angles will be from -0.4 radians to +0.4 radians (-22.9° to +22.9°).

3. A discrete-time system has a transfer function $H(z) = \dfrac{z}{z - 0.6}$. It is excited by a suddenly-applied sinusoid of the form $x[n] = 4\cos(2\pi n / 16)u[n]$. After a long time ($n \to \infty$) the response of the system approaches a sinusoid of the form $y[n] = A\cos\left(\dfrac{2\pi n}{B} + C\right)$. Using the result derived in the text

$$y[n] = \mathcal{Z}^{-1}\left(z\frac{N_1(z)}{D(z)}\right) + |H(p_1)|\cos\left(\Omega_0 n + \quad H(p_1)\right)u[n] \text{ where } p_1 = e^{j\Omega_0},$$

what are the numerical values of *A*, *B* and *C*? *A* = 1.9946 , *B* = 16 , *C* = -0.4757

$$\Omega_0 = \pi / 8 \Rightarrow p_1 = e^{j\pi/8}$$

$$H(p_1) = H(e^{j\pi/8}) = \frac{e^{j\pi/8}}{e^{j\pi/8} - 0.6} = \frac{0.9239 + j0.3827}{0.9239 + j0.3827 - 0.6}$$

$$= \frac{0.9239 + j0.3827}{0.3239 + j0.3827} = 1.7731 - j0.9135 = 1.9946 \quad -0.4757 = 1.9946e^{-j0.4757}$$

$$y[n] = 1.9946\cos\left(\frac{2\pi n}{16} - 0.4757\right)u[n]$$

4. A discrete-time bandpass filter has a transfer function $\mathrm{H}(z)=\dfrac{(z-1)(z+1)}{z^2-1.4z+0.98}$.

(a) Letting $z=e^{j\Omega}$ where $\Omega$ is real radian frequency, what two numerical values of $\Omega$ in the range $-\pi \le \Omega < \pi$ make the magnitude of the transfer function a maximum?

$\Omega$ = $\underline{-\pi/4 \text{ or } -0.785}$ and $\underline{\pi/4 \text{ or } 0.785}$

This transfer function has two zeros at $z=\pm 1$ and two poles at $z=0.9899e^{\pm j0.7854}$. These two poles are very close to the unit circle and the maximum response magnitude occurs when $z$ is closest to the poles. That occurs at an angle of $\pm\pi/4$ radians and those are the same as the values of $\Omega$ for a maximum magnitude.

(b) Letting $z=e^{j\Omega}$ where $\Omega$ is real radian frequency, what two numerical values of $\Omega$ in the range $-\pi \le \Omega < \pi$ make the magnitude of the transfer function a minimum?

$\Omega$ = $\underline{-\pi}$ and $\underline{0}$

The minimum magnitude occurs at the zeros which are at $z=\pm 1$.

$z=\pm 1 \Rightarrow \Omega=-\pi$ and $0$