Solution of ECE 316 Test #8 S03 3/12/03

1. Using the initial value and final value theorems find $\mathrm{x}[0]$ and $\lim_{n\to\infty}\mathrm{x}[n]$ for each signal, $\mathrm{x}[n]$, which has a unilateral z transform, $\mathrm{X}(z)$ below. If the final value theorem does not apply, just write "NA" (not applicable).

(a) $\mathrm{X}(z)=\dfrac{z}{z-\frac{2}{3}}$, $\mathrm{x}[0]=\lim_{z\to\infty}\dfrac{z}{z-\frac{2}{3}}=1$, $\lim_{n\to\infty}\mathrm{x}[n]=\lim_{z\to 1}(z-1)\dfrac{z}{z-\frac{2}{3}}=0$

(b) $\mathrm{X}(z)=\dfrac{z}{\left(z-\frac{2}{3}\right)(z-1)}$, $\mathrm{x}[0]=\lim_{z\to\infty}\dfrac{z}{\left(z-\frac{2}{3}\right)(z-1)}=0$, $\lim_{n\to\infty}\mathrm{x}[n]=\lim_{z\to 1}(z-1)\dfrac{z}{\left(z-\frac{2}{3}\right)(z-1)}=3$

2. If the unilateral z transform of $\mathrm{x}[n]$ is $\mathrm{X}(z)=\dfrac{z}{z-1}$, what are the z transforms of $\mathrm{x}[n-1]$ and $\mathrm{x}[n+1]$?

$$\mathrm{x}[n-1]\xleftrightarrow{z} z^{-1}\,\mathrm{X}(z)=z^{-1}\frac{z}{z-1}=\frac{1}{z-1}$$

$$\mathrm{x}[n+1]\xleftrightarrow{z} = z[\mathrm{X}(z)-\mathrm{x}[0]]=z\left[\frac{z}{z-1}-\mathrm{x}[0]\right] \qquad \mathrm{x}[n]=\mathrm{u}[n]\Rightarrow \mathrm{x}[0]=\mathrm{u}[0]=1$$

$$\mathrm{x}[n+1]\xleftrightarrow{z} = z\left[\frac{z}{z-1}-1\right]=\frac{z^2}{z-1}-z=\frac{z^2-z^2+z}{z-1}=\frac{z}{z-1}$$

3. What is the region of convergence (ROC) of the unilateral z transform of $\mathrm{x}[n]=\dfrac{-10z}{z+\frac{1}{2}}$?

ROC is $|z|>\dfrac{1}{2}$