

Solution to ECE Test 9 S09

1. Find the numerical values of A , a and b .

$$Aa^n \sin(bn)u[n] \xleftrightarrow{\mathcal{F}} \frac{z}{z^2 + 0.81}$$

$$\frac{z}{z^2 + 0.81} = \frac{1}{\alpha \sin(\Omega_0)} \frac{z\alpha \sin(\Omega_0)}{z^2 - 2\alpha z \cos(\Omega_0) + 0.81} \Rightarrow \alpha = 0.9, \Omega_0 = \pi/2$$

$$1.111(0.9)^n \sin(\pi n/2)u[n] \xleftrightarrow{\mathcal{F}} \frac{z}{z^2 + 0.81} = 1.111 \frac{0.9z}{z^2 + 0.81}$$

2. If $x[n] \xleftrightarrow{\mathcal{F}} X(z)$ and $X(z) = \frac{5z^2 - 2z + 3}{3z^3 + 2z^2 - 3z + 1}$ what is the numerical value of $x[1]$?

$$\frac{5z^2 - 2z + 3}{3z^3 + 2z^2 - 3z + 1} \xrightarrow{z=1} \frac{5/3z}{5z^2 + 10z/3 - 5 + 5/3z} \Rightarrow x[1] = 5/3$$

3. If $X(z) = \frac{z(z-0.5)(z+0.3)}{(z-0.8)(z+0.1)(z-1)}$ and $x[n] \xleftrightarrow{\mathcal{F}} X(z)$, what is the numerical final value of $x[n]$?

$$\lim_{n \rightarrow \infty} x[n] = \lim_{z \rightarrow 1} (z-1)X(z) = \lim_{z \rightarrow 1} \frac{z(z-0.5)(z+0.3)}{(z-0.8)(z+0.1)} = \frac{1(0.5)(1.3)}{(0.2)(1.1)} = 2.955$$

Solution to ECE Test 9 S09

1. Find the numerical values of A , a and b .

$$Aa^n \sin(bn)u[n] \xleftrightarrow{\mathcal{Z}} \frac{z}{z^2 + 0.49}$$

$$\frac{z}{z^2 + 0.49} = \frac{1}{\alpha \sin(\Omega_0)} \frac{z\alpha \sin(\Omega_0)}{z^2 - 2\alpha z \cos(\Omega_0) + 0.81} \Rightarrow \alpha = 0.7, \Omega_0 = \pi/2$$

$$1.429(0.7)^n \sin(\pi n/2)u[n] \xleftrightarrow{\mathcal{Z}} \frac{z}{z^2 + 0.49} = 1.429 \frac{0.7z}{z^2 + 0.49}$$

2. If $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$ and $X(z) = \frac{2z^2 - 2z + 3}{3z^3 + 2z^2 - 3z + 1}$ what is the numerical value of $x[1]$?

$$3z^3 + 2z^2 - 3z + 1 \overline{) \frac{2z^2 - 2z + 3}{2z^2 + 4z/3 - 2 + 2/3z}} \Rightarrow x[1] = 2/3$$

3. If $X(z) = \frac{z(z-0.4)(z+0.2)}{(z-0.8)(z+0.1)(z-1)}$ and $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, what is the numerical final value of $x[n]$?

$$\lim_{n \rightarrow \infty} x[n] = \lim_{z \rightarrow 1} (z-1)X(z) = \lim_{z \rightarrow 1} \frac{z(z-0.4)(z+0.2)}{(z-0.8)(z+0.1)} = \frac{1(0.6)(1.2)}{(0.2)(1.1)} = 3.273$$

Solution to ECE Test 9 S09

1. Find the numerical values of A , a and b .

$$Aa^n \sin(bn)u[n] \xleftrightarrow{\mathcal{Z}} \frac{z}{z^2 + 0.64}$$

$$\frac{z}{z^2 + 0.64} = \frac{1}{\alpha \sin(\Omega_0)} \frac{z\alpha \sin(\Omega_0)}{z^2 - 2\alpha z \cos(\Omega_0) + 0.64} \Rightarrow \alpha = 0.8, \Omega_0 = \pi/2$$

$$1.25(0.8)^n \sin(\pi n/2)u[n] \xleftrightarrow{\mathcal{Z}} \frac{z}{z^2 + 0.64} = 1.25 \frac{0.8z}{z^2 + 0.64}$$

2. If $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$ and $X(z) = \frac{z^2 - 2z + 3}{3z^3 + 2z^2 - 3z + 1}$ what is the numerical value of $x[1]$?

$$3z^3 + 2z^2 - 3z + 1 \left) \frac{z^2 - 2z + 3}{z^2 - 2z + 3} \frac{1/3z}{z^2 + 2z/3 - 1 + 1/3z} \Rightarrow x[1] = 1/3$$

3. If $X(z) = \frac{z(z-0.2)(z+0.1)}{(z-0.8)(z+0.3)(z-1)}$ and $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, what is the numerical final value of $x[n]$?

$$\lim_{n \rightarrow \infty} x[n] = \lim_{z \rightarrow 1} (z-1)X(z) = \lim_{z \rightarrow 1} \frac{z(z-0.2)(z+0.1)}{(z-0.8)(z+0.3)} = \frac{1(0.8)(1.1)}{(0.2)(1.3)} = 3.385$$