Solution of ECE 316 Test #9 S03 3/26/03

1. If $\mathrm{H}(z)=\dfrac{z^2}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{3}\right)}$ then, by finding the partial fraction expansion of this improper fraction in *z* two different ways, its inverse *z* transform, $\mathrm{h}[n]$ can be written in two different forms,

$$\mathrm{h}[n]=\left[A\left(\frac{1}{2}\right)^n+B\left(-\frac{1}{3}\right)^n\right]\mathrm{u}[n] \text{ and } \mathrm{h}[n]=\delta[n]+\left[C\left(\frac{1}{2}\right)^{n-1}+D\left(-\frac{1}{3}\right)^{n-1}\right]\mathrm{u}[n-1] \ .$$

Find *A*, *B*, *C*, and *D* (2 pts each).

$$\frac{\mathrm{H}(z)}{z}=\frac{z}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{3}\right)}=\frac{\frac{3}{5}}{z-\frac{1}{2}}+\frac{\frac{2}{5}}{z+\frac{1}{3}}\Rightarrow \mathrm{H}(z)=\frac{\frac{3}{5}z}{z-\frac{1}{2}}+\frac{\frac{2}{5}z}{z+\frac{1}{3}}\Rightarrow \mathrm{h}[n]=\left[\frac{3}{5}\left(\frac{1}{2}\right)^n+\frac{2}{5}\left(-\frac{1}{3}\right)^n\right]\mathrm{u}[n]$$

$$A=\frac{3}{5}=0.6 \ , \ B=\frac{2}{5}=0.4$$

$$\mathrm{H}(z)=\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{3}\right)}=\frac{z^2}{z^2-\frac{1}{6}z-\frac{1}{6}}=1+\frac{1}{6}\frac{z+1}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{3}\right)}=1+\frac{1}{6}\left[\frac{\frac{9}{5}}{z-\frac{1}{2}}-\frac{\frac{4}{5}}{z+\frac{1}{3}}\right]$$

$$\mathrm{h}[n]=\delta[n]+\left[\frac{3}{10}\left(\frac{1}{2}\right)^{n-1}-\frac{2}{15}\left(-\frac{1}{3}\right)^{n-1}\right]\mathrm{u}[n-1]$$

$$C=\frac{3}{10}=0.3 \ , \ D=-\frac{2}{15}=-0.1333\ldots$$

Circle two correct answers.

2. (2 pts) Which of these lines in the *s* plane maps into the unit circle in the *z* plane?

The ω axis and The line segment $0<\omega<\dfrac{2\pi}{T_s},\sigma=0$