Solution to ECE Test #10 S09

1. By using two different partial-fraction expansion methods, the inverse z transform of

$$\mathrm{X}(z) = \frac{z^2}{z^2 - (5/6)z + 1/6}$$

can be expressed in either of two forms

$$\mathrm{x}[n] = \left(Aa^n + Bb^n\right)\mathrm{u}[n] \text{ or } \mathrm{x}[n] = K\delta[n] + \left[Ca^{n-1} + Db^{n-1}\right]\mathrm{u}[n-1] \text{ .}$$

Find the numerical values of A , a, B, b, K, C, and D.

A = <u>3</u> , a = <u>1/2</u> , B = <u>-2</u> , b = <u>1/3</u> , K = <u>1</u> , C = <u>3/2</u> , D = -<u>2/3</u>

$$\frac{\mathrm{X}(z)}{z} = \frac{z}{(z-1/2)(z-1/3)} = \frac{3}{z-1/2} - \frac{2}{z-1/3}$$

$$\mathrm{X}(z) = \frac{3z}{z-1/2} - \frac{2z}{z-1/3} \Rightarrow \mathrm{x}[n] = \left[3(1/2)^n - 2(1/3)^n\right]\mathrm{u}[n]$$

$$\mathrm{X}(z) = 1 + \frac{(5/6)z - 1/6}{z^2 - (5/6)z + 1/6} = 1 + \left[\frac{3/2}{z-1/2} - \frac{2/3}{z-1/3}\right]$$

$$\mathrm{x}[n] = \delta[n] + \left[(3/2)(1/2)^{n-1} - (2/3)(1/3)^{n-1}\right]\mathrm{u}[n-1]$$

2. Using the mapping relationship $z = e^{sT_s}$ find the numerical real and imaginary parts of the z-plane location corresponding to these s-plane points with the specified sampling rates $f_s = 1/T_s$.

(a) $\sigma = -1$, $\omega = 4\pi$ with $f_s = 8$. z = 0 + j 0.8825

(b) $\sigma = -1$, $\omega = 12\pi$ with $f_s = 4$. z = -0.7788 + j 0

(c) $\sigma \to -\infty$, $\omega = 0$ with $f_s = 100$. z = 0 + j 0

Solution to ECE Test #10 S09

1. By using two different partial-fraction expansion methods, the inverse z transform of

$$X(z) = \frac{z^2}{z^2 - (7/12)z + 1/12}$$

can be expressed in either of two forms

$$x[n] = \left(Aa^n + Bb^n\right)u[n] \text{ or } x[n] = K\delta[n] + \left[Ca^{n-1} + Db^{n-1}\right]u[n-1] .$$

Find the numerical values of A , a, B, b, K, C, and D.

A = 4 , a = 1/3 , B = -3 , b = 1/4 , K = 1 , C = 4/3 , D = -3/4

$$\frac{X(z)}{z} = \frac{z}{(z-1/3)(z-1/4)} = \frac{4}{z-1/3} - \frac{3}{z-1/4}$$

$$X(z) = \frac{4z}{z-1/3} - \frac{3z}{z-1/4} \Rightarrow x[n] = \left[4(1/3)^n - 3(1/4)^n\right]u[n]$$

$$X(z) = 1 + \frac{(7/12)z - 1/12}{z^2 - (7/12)z + 1/12} = 1 + \left[\frac{4/3}{z-1/3} - \frac{3/4}{z-1/4}\right]$$

$$x[n] = \delta[n] + \left[(4/3)(1/3)^{n-1} - (3/4)(1/4)^{n-1}\right]u[n-1]$$

2. Using the mapping relationship $z = e^{sT_s}$ find the numerical real and imaginary parts of the z-plane location corresponding to these s-plane points with the specified sampling rates $f_s = 1/T_s$.

(a) $\sigma = -1$, $\omega = 4\pi$ with $f_s = 8$. $z = 0 + j\ 0.8825$

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