Solution of ECE 316 Test 10 S06

1. $\left[Aa^n + Bb^n\right]\mathrm{u}\left[n\right] \overset{z}{\longleftrightarrow} \dfrac{z^2}{\left(z+0.2\right)\left(z-0.5\right)}$

$A = 2/7$, $a = -0.2$, $B = 5/7$, $b = 0.5$

$$\frac{z^2}{\left(z+0.2\right)\left(z-0.5\right)} = z\left[\frac{2/7}{z+0.2} + \frac{5/7}{z-0.5}\right] = \frac{2z/7}{z+0.2} + \frac{5z/7}{z-0.5}$$

$$\left[\left(2/7\right)\left(-0.2\right)^n + \left(5/7\right)\left(0.5\right)^n\right]\mathrm{u}\left[n\right] \overset{z}{\longleftrightarrow} \frac{z^2}{\left(z+0.2\right)\left(z-0.5\right)}$$

2. $\alpha^n\left[A\cos\left(\Omega_0 n\right) + B\sin\left(\Omega_0 n\right)\right]\mathrm{u}\left[n\right] \overset{z}{\longleftrightarrow} \dfrac{z^2}{z^2 - 1.8z + 0.95}$

$\alpha = 0.975$, $A = 1$, $B = 2.405$, $\Omega_0 = 0.394$

$$\frac{z^2}{z^2 - 1.8z + 0.95} \Rightarrow \alpha = \sqrt{0.95} = 0.975 \Rightarrow 1.8 = 2\left(0.975\right)\cos\left(\Omega_0\right) \Rightarrow \Omega_0 = 0.395$$

$$\frac{z^2}{z^2 - 1.8z + 0.95} = \frac{z^2 - \alpha z\cos\left(\Omega_0\right)}{z^2 - 1.8z + 0.95} + \frac{\alpha z\cos\left(\Omega_0\right)}{z^2 - 1.8z + 0.95}$$

$$\frac{z^2}{z^2 - 1.8z + 0.95} = \frac{z^2 - \alpha z\cos\left(\Omega_0\right)}{z^2 - 1.8z + 0.95} + \frac{\cos\left(\Omega_0\right)}{\sin\left(\Omega_0\right)}\frac{\alpha z\sin\left(\Omega_0\right)}{z^2 - 1.8z + 0.95}$$

$$\frac{z^2}{z^2 - 1.8z + 0.95} = \frac{z^2 - 0.9z}{z^2 - 1.8z + 0.95} + 2.405\frac{0.3742z}{z^2 - 1.8z + 0.95}$$

$$\left(0.975\right)^n\left[\cos\left(0.394n\right) + 2.405\sin\left(0.394n\right)\right]\mathrm{u}\left[n\right] \overset{z}{\longleftrightarrow} \frac{z^2}{z^2 - 1.8z + 0.95}$$

Solution of ECE 316 Test 10 S06

1. $\left[Aa^n + Bb^n\right]\mathrm{u}\left[n\right] \overset{\mathrm{z}}{\longleftrightarrow} \dfrac{z^2}{\left(z-0.3\right)\left(z+0.1\right)}$

$A = 3/4$, $a = 0.3$, $B = 1/4$, $b = -0.1$

$$\frac{z^2}{\left(z-0.3\right)\left(z+0.1\right)} = z\left[\frac{3/4}{z-0.3} + \frac{1/4}{z+0.1}\right] = \frac{3z/4}{z-0.3} + \frac{z/4}{z+0.1}$$

$$\left[\left(3/4\right)\left(0.3\right)^n + \left(1/4\right)\left(-0.1\right)^n\right]\mathrm{u}\left[n\right] \overset{\mathrm{z}}{\longleftrightarrow} \frac{z^2}{\left(z-0.3\right)\left(z+0.1\right)}$$

2. $\alpha^n\left[A\cos\left(\Omega_0 n\right) + B\sin\left(\Omega_0 n\right)\right]\mathrm{u}\left[n\right] \overset{\mathrm{z}}{\longleftrightarrow} \dfrac{z^2}{z^2 - 1.6z + 0.9}$

$\alpha = 0.949$, $A = 1$, $B = 1.567$, $\Omega_0 = 0.568$

$$\frac{z^2}{z^2 - 1.6z + 0.9} \Rightarrow \alpha = \sqrt{0.9} = 0.949 \Rightarrow 1.6 = 2\left(0.949\right)\cos\left(\Omega_0\right) \Rightarrow \Omega_0 = 0.568$$

$$\frac{z^2}{z^2 - 1.6z + 0.9} = \frac{z^2 - \alpha z\cos\left(\Omega_0\right)}{z^2 - 1.6z + 0.9} + \frac{\alpha z\cos\left(\Omega_0\right)}{z^2 - 1.6z + 0.9}$$

$$\frac{z^2}{z^2 - 1.6z + 0.9} = \frac{z^2 - \alpha z\cos\left(\Omega_0\right)}{z^2 - 1.6z + 0.9} + \frac{\cos\left(\Omega_0\right)}{\sin\left(\Omega_0\right)}\frac{\alpha z\sin\left(\Omega_0\right)}{z^2 - 1.6z + 0.9}$$

$$\frac{z^2}{z^2 - 1.6z + 0.9} = \frac{z^2 - 0.8z}{z^2 - 1.6z + 0.9} + 1.567\frac{0.538z}{z^2 - 1.6z + 0.9}$$

$$\left(0.949\right)^n\left[\cos\left(0.568n\right) + 1.567\sin\left(0.568n\right)\right]\mathrm{u}\left[n\right] \overset{\mathrm{z}}{\longleftrightarrow} \frac{z^2}{z^2 - 1.6z + 0.9}$$